

Binary black hole remnants of first stars for the gravitational wave source

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Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**

(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21} . It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ . The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_{\odot}$ and $29^{+4}_{-4} M_{\odot}$, and the final black hole mass is $62^{+4}_{-4} M_{\odot}$, with $3.0^{+0.5}_{-0.5} M_{\odot} c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

- $36M_{\odot} + 29M_{\odot}$, circular orbit

GW150914

- More than factor 2 larger mass of BH compared with that in X-ray binary
- Many theories exist such as
 - 1) PopII BBH
 - 2) PopIII BBH **Low metal field binaries**
 - 3) Primordial Binary BH(PBBH)
 - 4) Three body origin from Globular Cluster
 - 5) Fragmentation of very massive stars
 -

Why field binaries?

- There are many massive close binaries

Example

Milky way young open clusters

71 O stars fbinary=69+/-9% (P<3200days) Sana et al. 2012

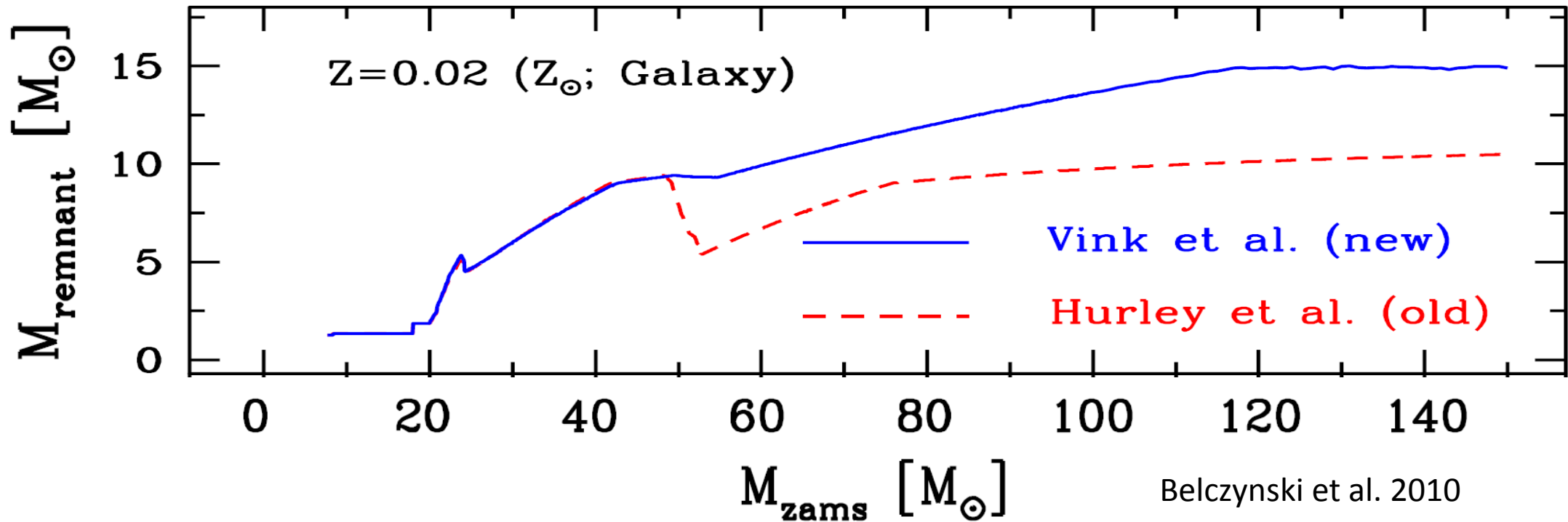
30 Doradus (Tarantula Nebula)

362 O stars fbinary=51+/-4%(P<3200days) Sana et al. 2013

- Field binaries tend to become circular orbit by binary interactions

Why low metal?

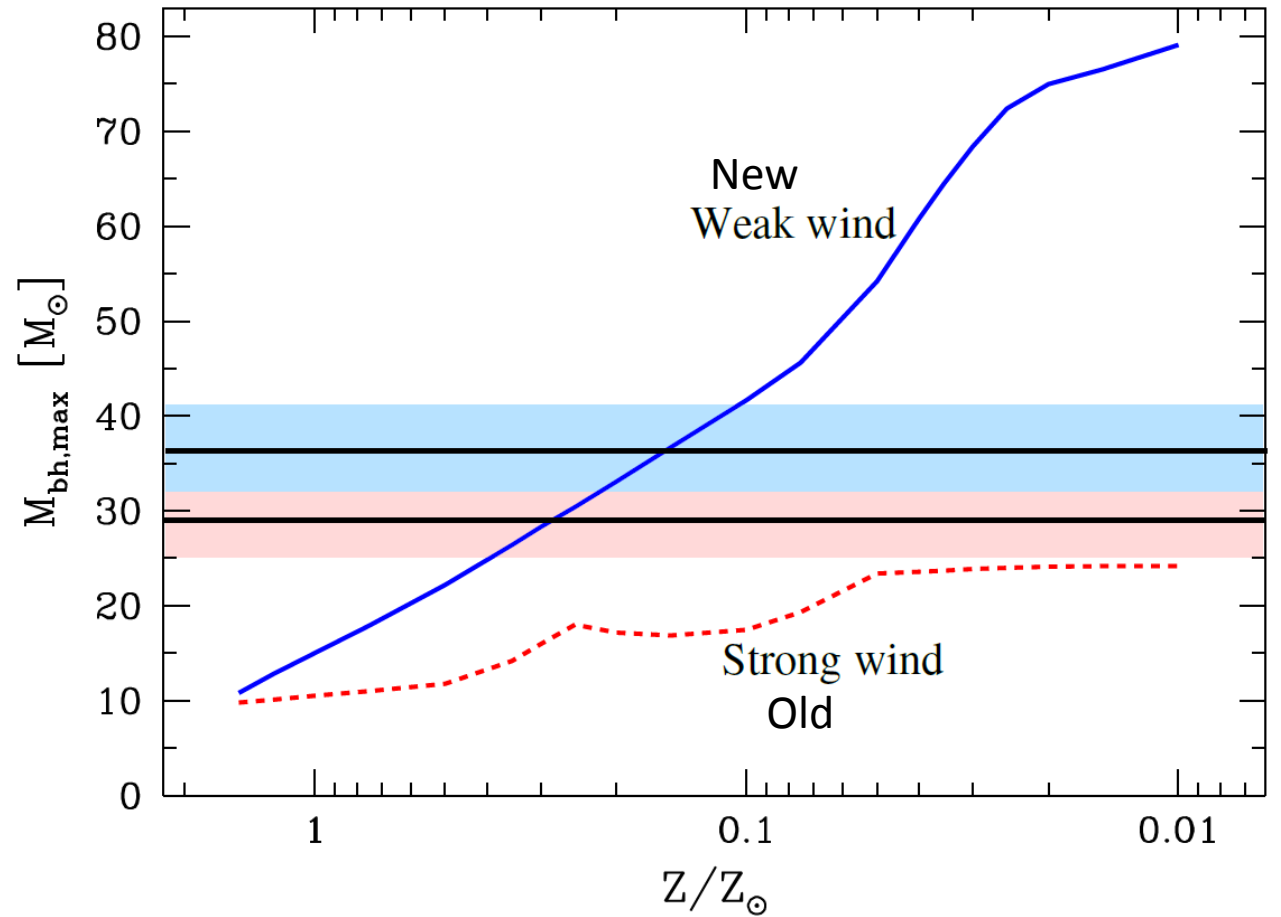
- If the progenitor of BH is Pop I (=Solar metal stars)



- Orbit become wide due to wind mass loss

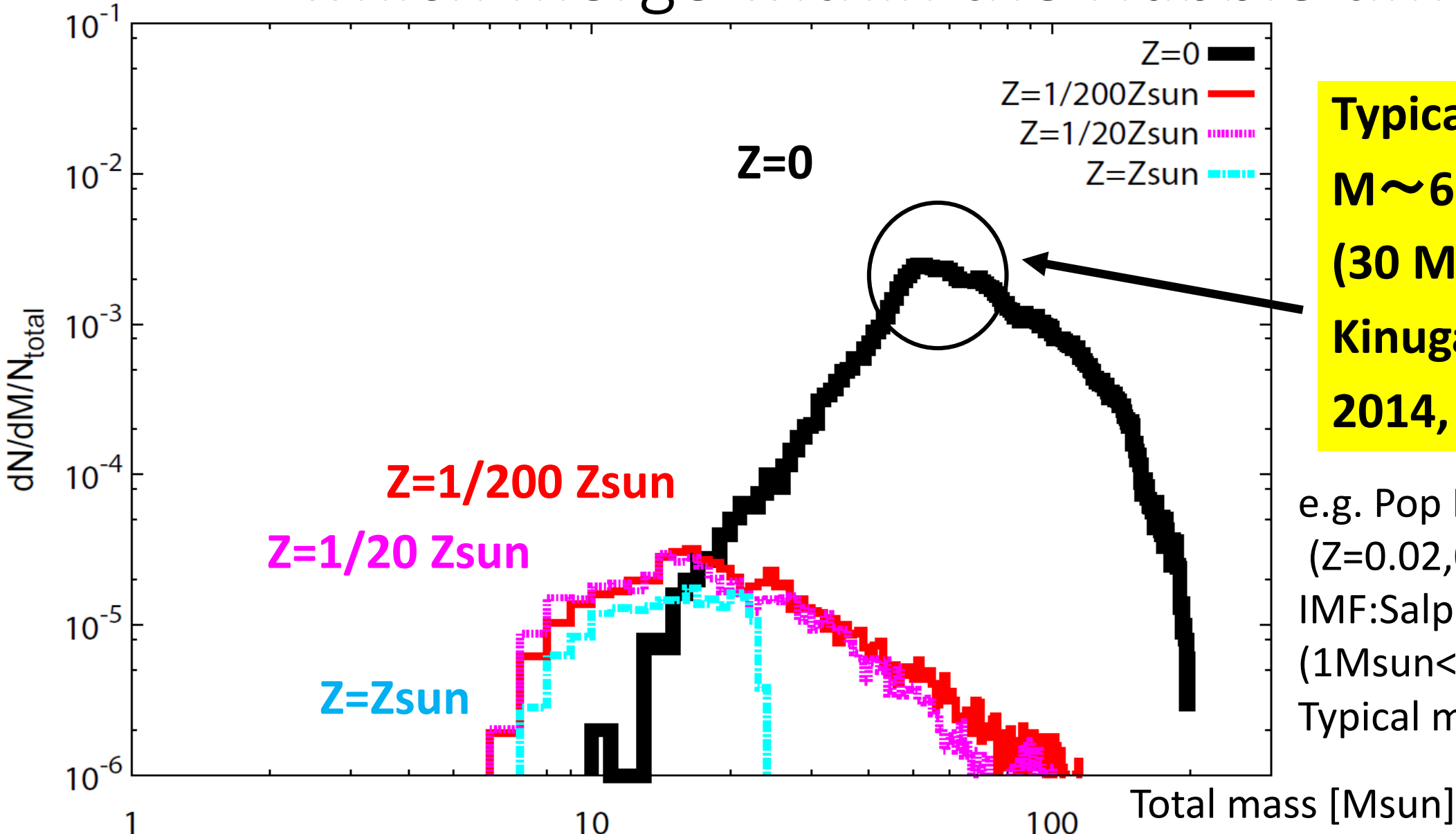
Why low metal?

- If the progenitor is low metal,
- Pop II ($Z < 0.1 Z_{\text{sun}}$)
Typical mass is same as Pop I
But, weak wind mass loss
- Pop III (No metal)
Pop III stars are the first stars after the Big Bang.
Typical mass is more massive than Pop I, II
 $M_{\text{pop III}} \sim 10\text{--}100 M_{\text{sun}}$
No wind mass loss due to no metal.



Initial: $8 M_{\text{sun}} < M < 150 M_{\text{sun}}$
Single stellar evolution
with 2 stellar wind models.
(Belczynski et al. 2010,
Abbot et al. 2016)

Total mass distribution of BH-BH which merge within the Hubble time

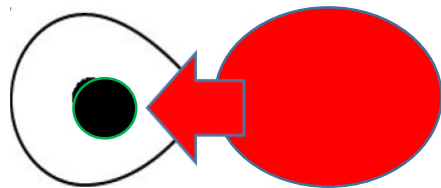


Typical total mass
 $M \sim 60 M_{\odot}$
 $(30 M_{\odot} + 30 M_{\odot})$
Kinugawa et al.
2014, 2016

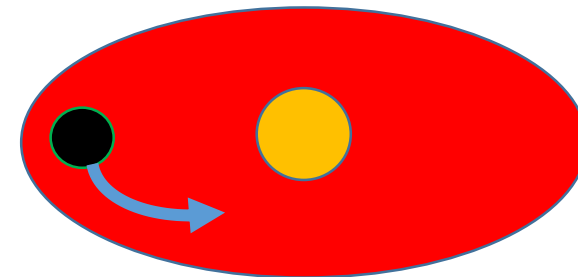
e.g. Pop I, Pop II
($Z=0.02, 0.001, 0.0001$)
IMF: Salpeter
($1 M_{\text{sun}} < M < 140 M_{\text{sun}}$)
Typical mass $\sim 10 M_{\odot}$

What do determine the BH-BH mass?

- Steller wind mass loss
- Binary interactions
(Mass transfer, Common envelope)



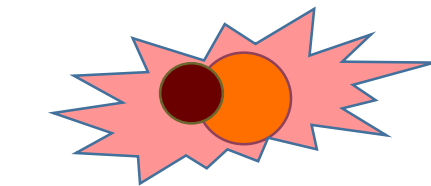
Mass transfer



Common envelope



Close binary



or merge

Why Pop III binaries become 30Msun BH-BH

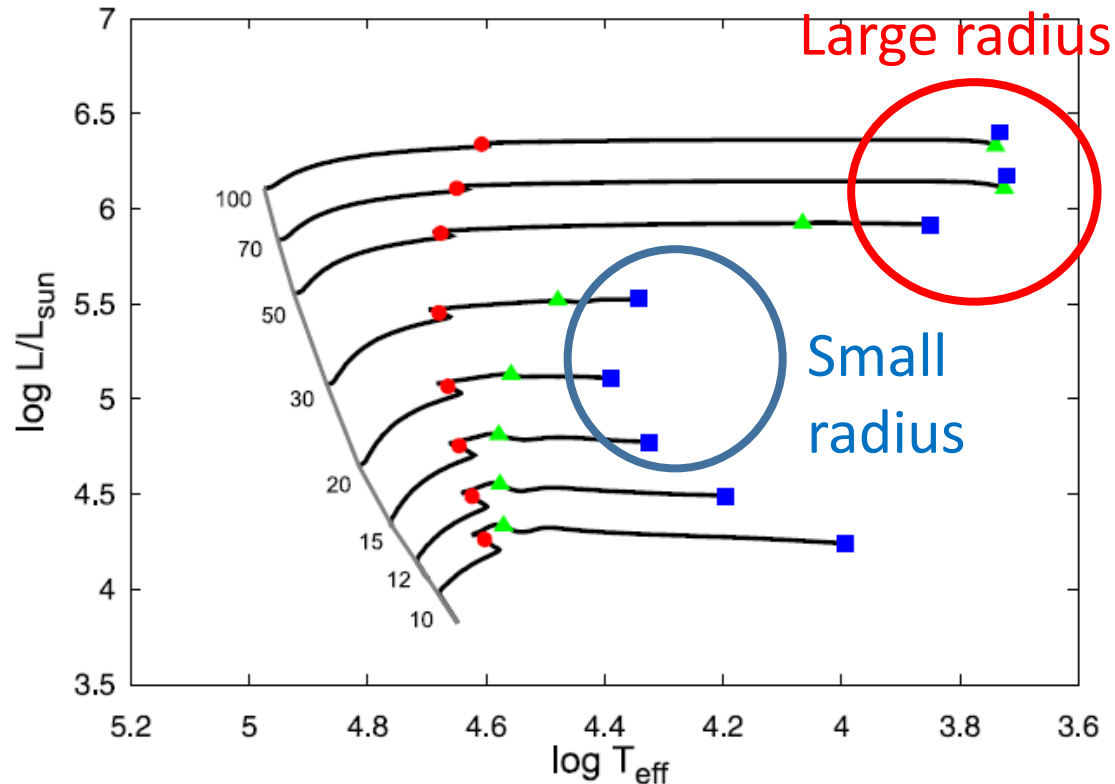
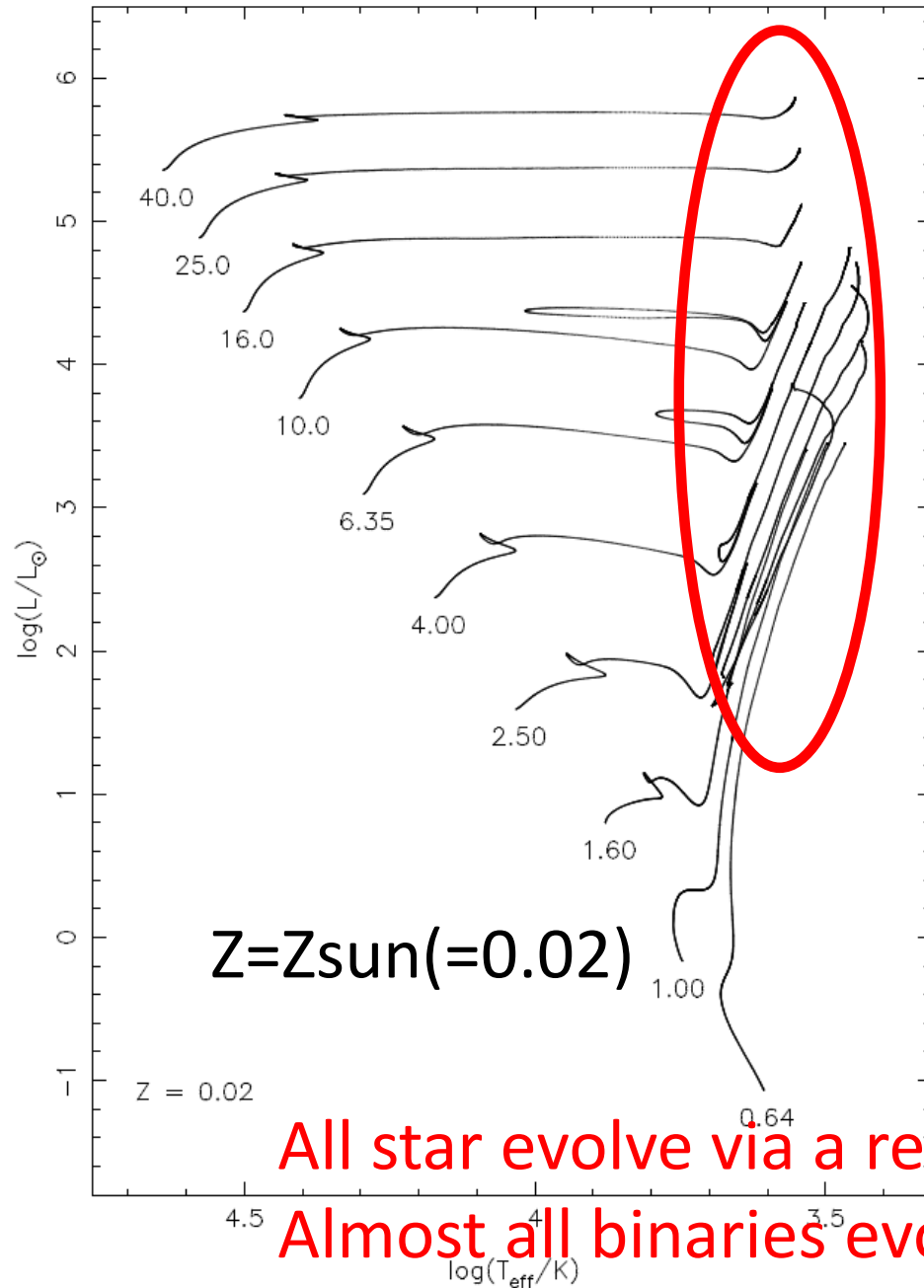


Figure 1. The Hertzsprung-Russell (HR) diagram for the Pop III stars of mass $10 M_{\odot} \leq M \leq 100 M_{\odot}$ using the data taken from [Marigo et al \(2001\)](#). The number attached to each solid curve is the mass of each star in unit of M_{\odot} . The dashed line shows the ZAMS (Zero Age Main Sequence) stars. Red circles, green triangles and blue squares correspond to the beginning of He-burning, the end of the He-burning and the beginning of the C-burning, respectively.

- $M > 50 M_{\text{sun}}$ red giant
 - Mass transfer is unstable
 - common envelope
 - 1/3~1/2 of initial mass (~25-30Msun)
- $M < 50 M_{\text{sun}}$ blue giant
 - Mass transfer is stable
 - mass loss is not so effective
 - 2/3~1 of initial mass (25-30Msun)



All star evolve via a red giant

Almost all binaries evolve via similar evolution pass

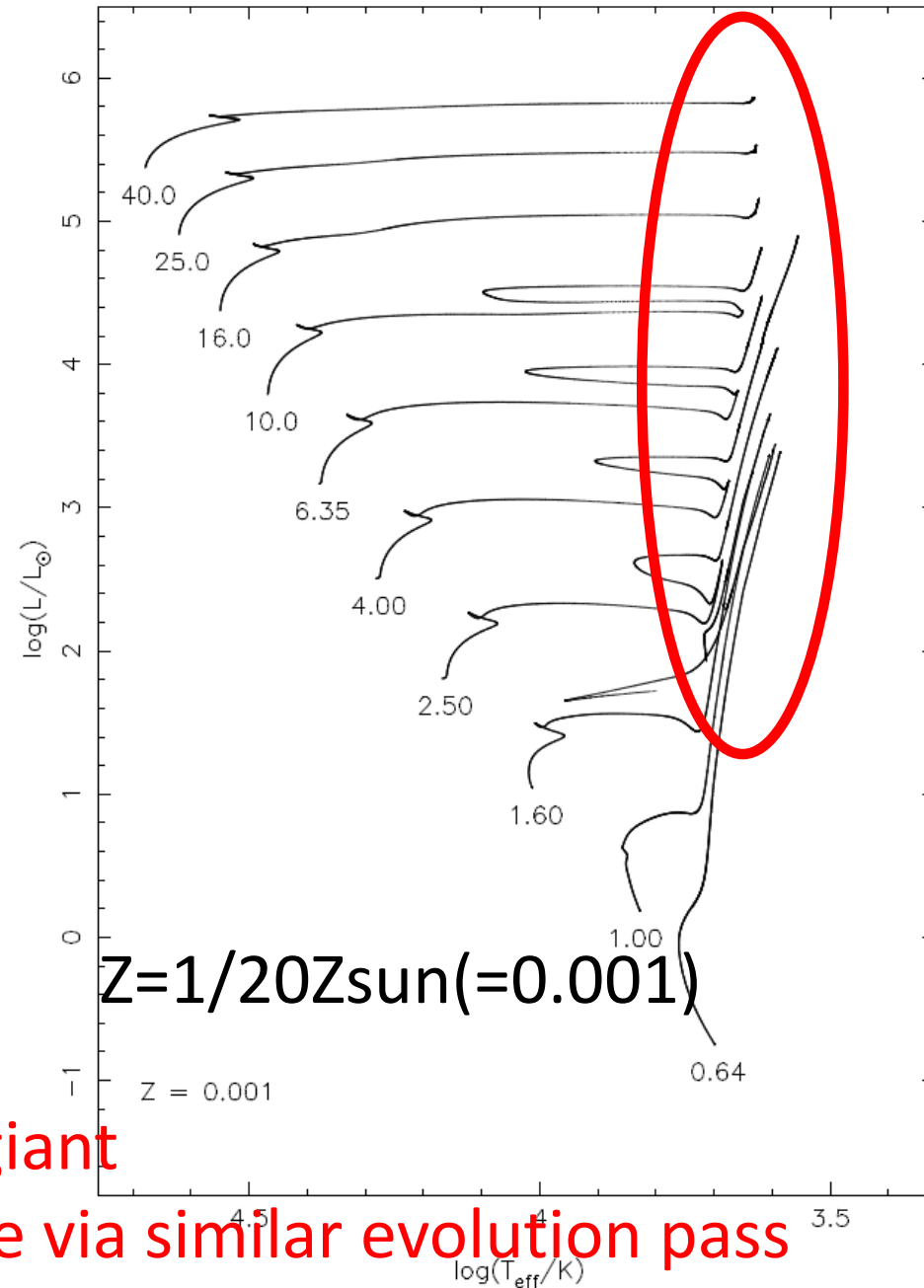
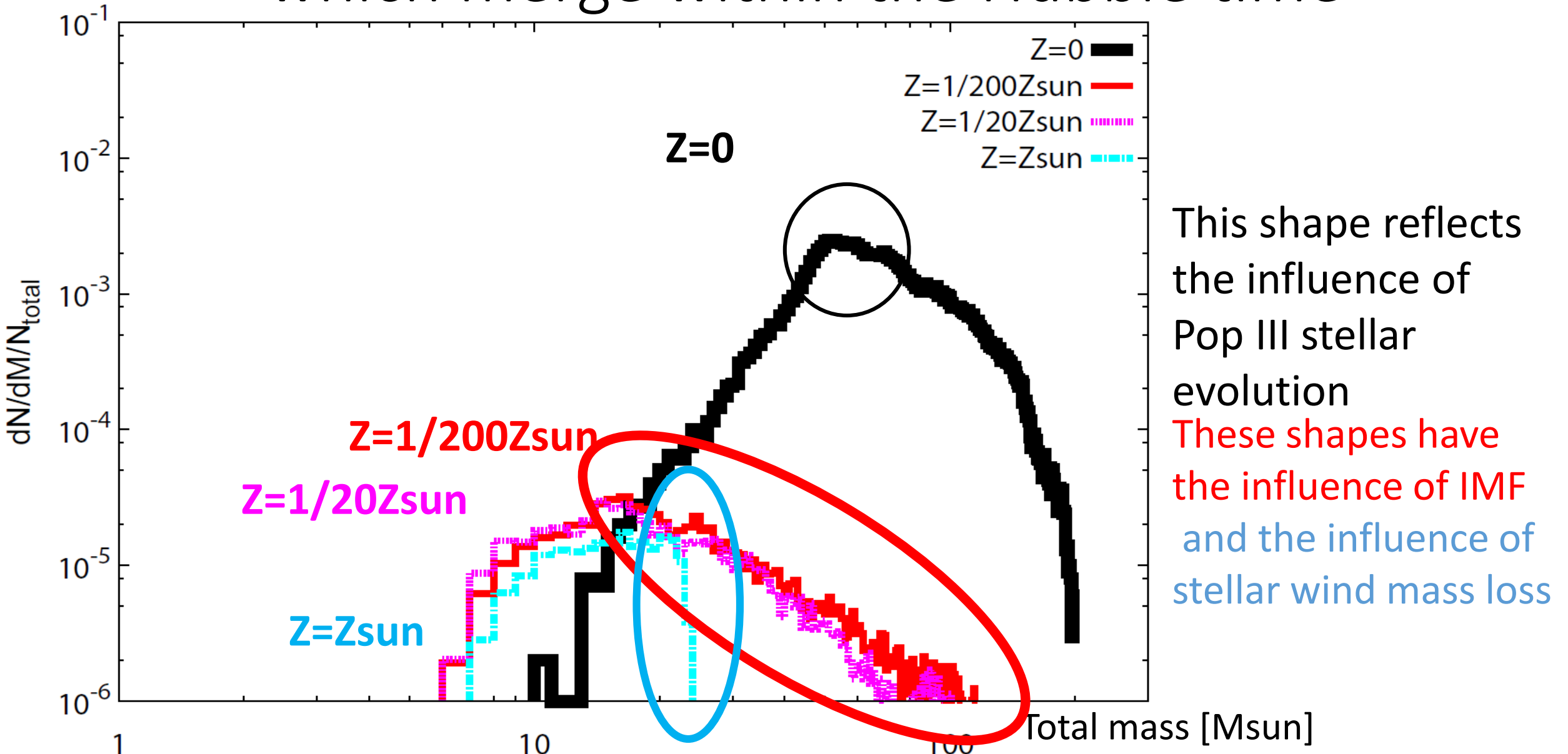


Figure 1. Selected OVS evolution tracks for $Z = 0.02$, for masses 0.64, 1.0, 1.6, 2.5, 4.0, 6.35, 10, 16, 25 and $40 M_{\odot}$.

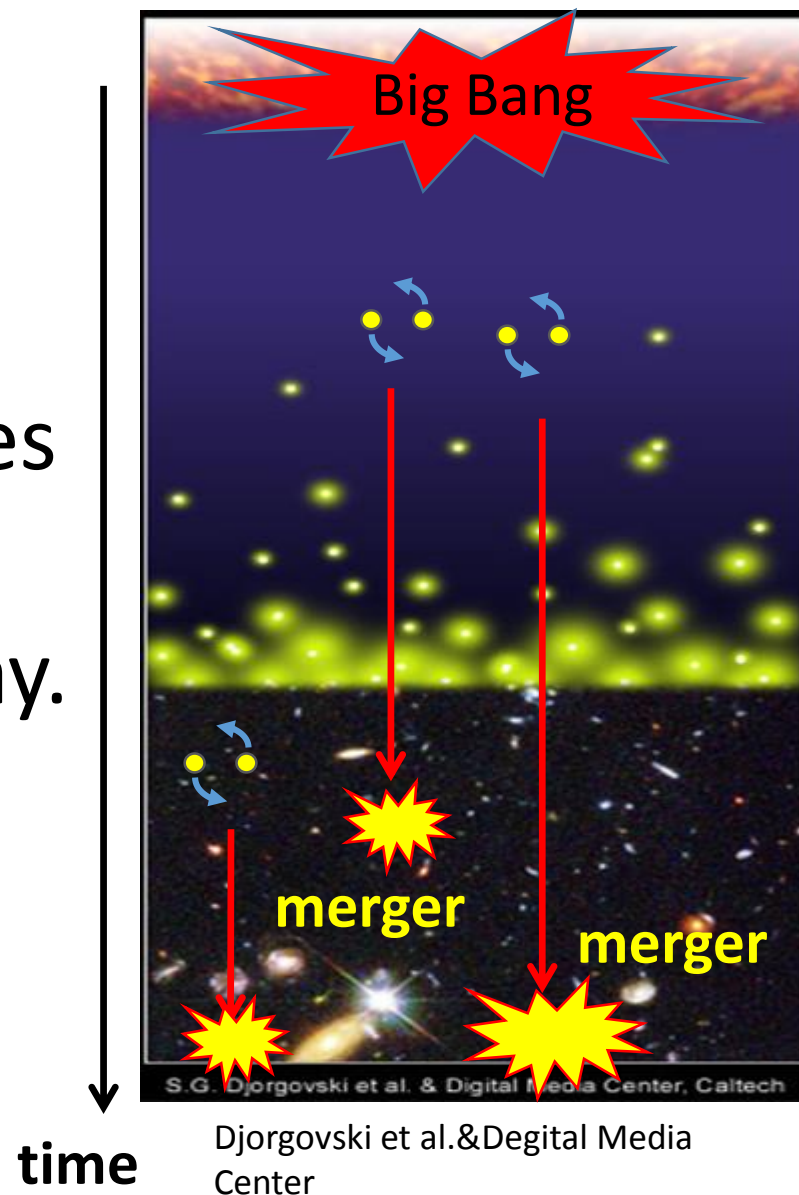
Figure 2. Same as Fig. 1 for $Z = 0.001$. The $1.0 M_{\odot}$ post He flash track has been omitted for clarity.

Total mass distribution of BBH which merge within the Hubble time



Pop III BBH remnants for gravitational wave

- Pop III stars were born and died at $z \sim 10$
- The typical merger time of compact binaries $\sim 10^{8-10}$ yr
- We might see Pop III BBH at the present day.



Pop III BBH?

ASTROPHYSICAL IMPLICATIONS OF THE BINARY BLACK-HOLE MERGER GW150914

ApJL Abbot. et al 2016

[2014](#), [Domolik et al. 2015](#)).

On the extreme low-metallicity end, it has been proposed that BBH formation is also possible in the case of stellar binaries at zero metallicity (Population III [PopIII] stars; see Belczynski et al. [2004](#); Kinugawa et al. [2014](#)). The predictions from these studies are even more uncertain, since we have no observational constraints on the properties of first-generation stellar binaries (e.g., mass function, mass ratios, orbital separations). However, if one assumes that the properties of PopIII massive binaries are not very different from binary populations in the local universe (admittedly a considerable extrapolation), then recently predicted BBH total masses agree astonishingly well with GW150914 and can have sufficiently long merger times to occur in the nearby universe (Kinugawa et al. [2014](#)). This is in contrast to the predicted mass properties

Detection rate of Pop III BH-BH

- **Detection rate of Pop III BBH (GW150914 like BBH) in our standard model**

$$R \sim 180 \left(\frac{SFR_{peak}}{10^{-2.5}} \right) \left(\frac{f_b/(1+f_b)}{0.33} \right) [\text{yr}^{-1}] (S/N > 8)$$

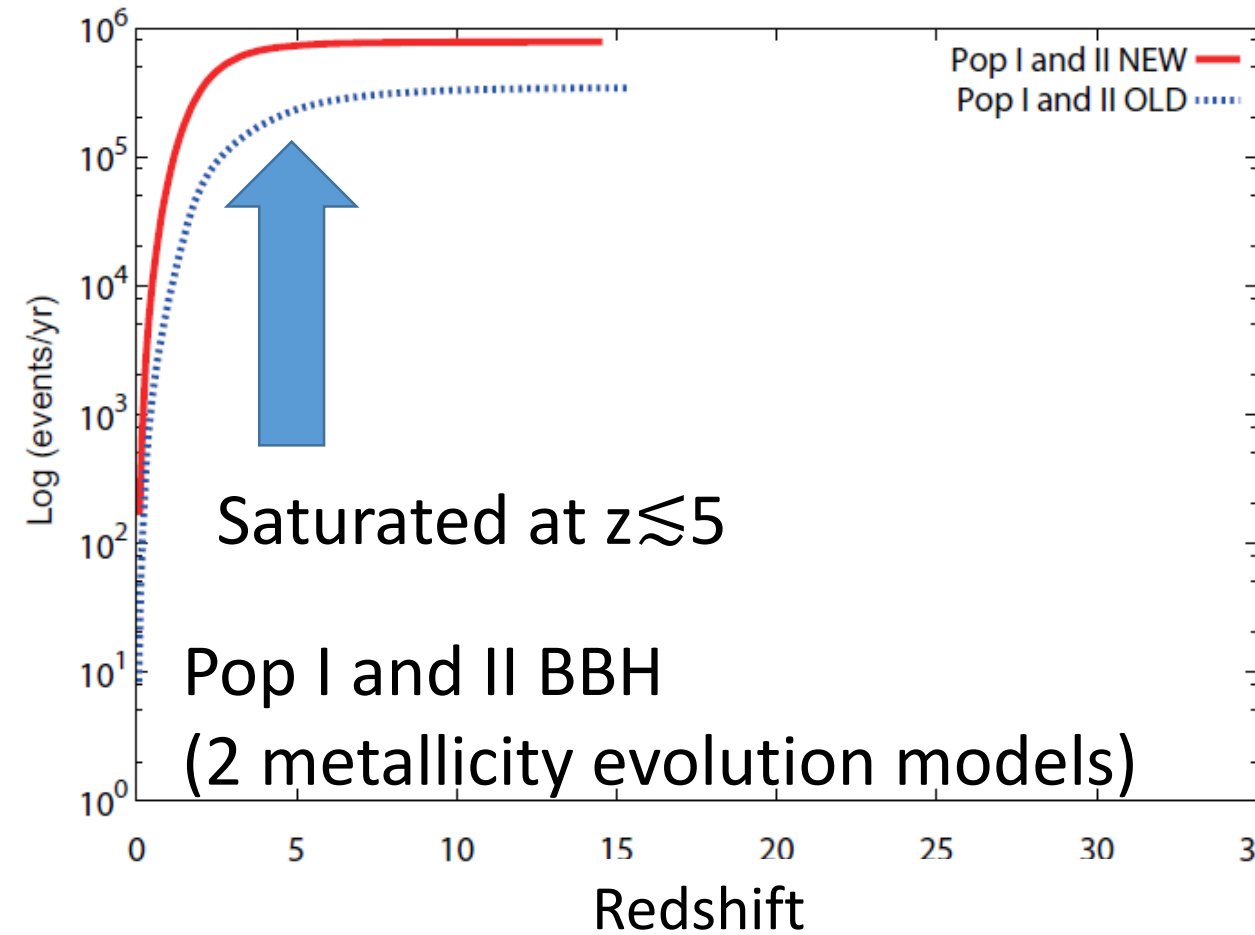
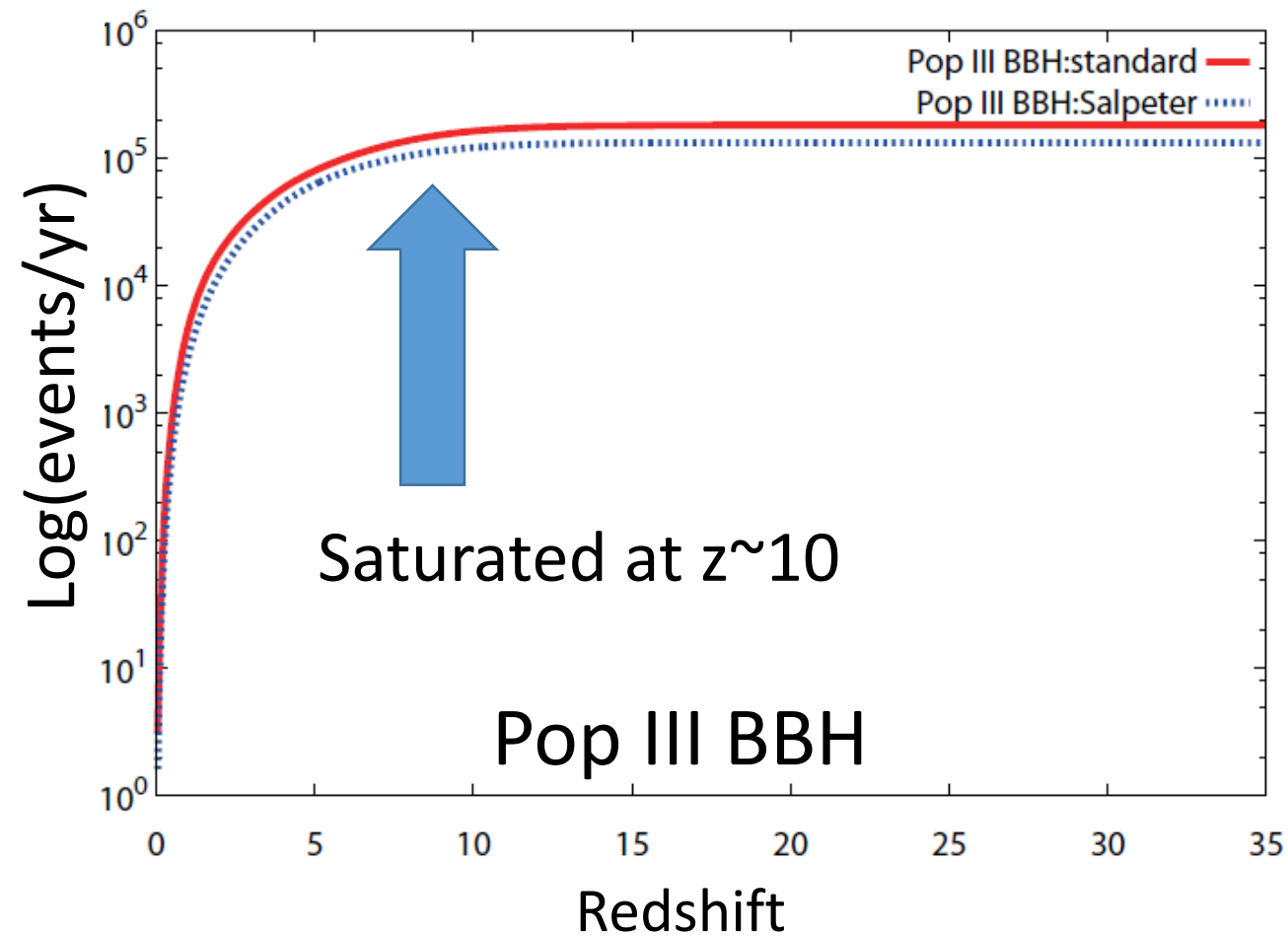
- Typical mass

$$M \sim 30 M_{\odot} \rightarrow \text{We can see QNM of merged BBH}$$

We might detect the Pop III BH-BH by GW

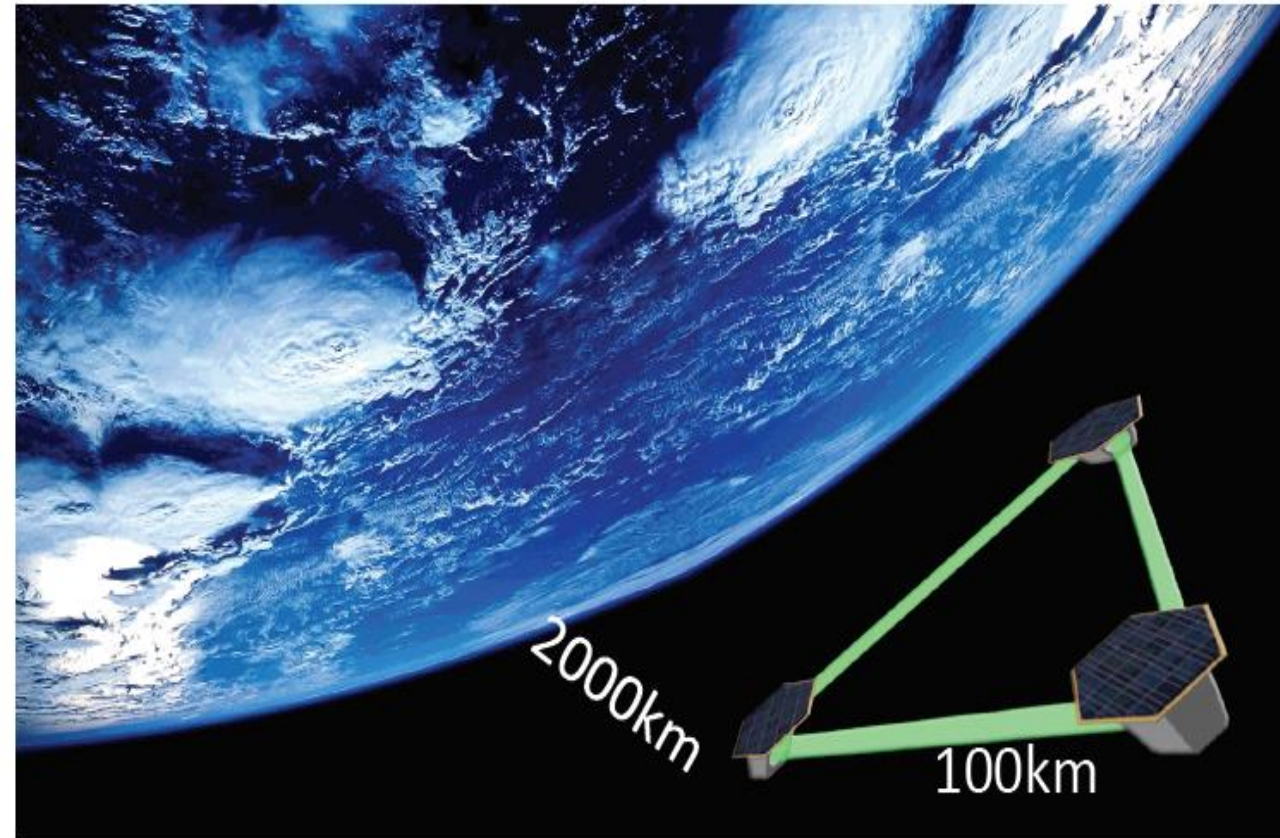
1. We might see BH QNM from Pop III BH-BH
→ We might check GR by Pop III BH QNM
2. The mass distribution might distinguish Pop III from Pop I, Pop II
→ The evidence of Pop III star

Cumulative BH-BH merger rate



future plan of GW observer : pre-DECIGO and DECIGO

- DECIGO: Japanese space gravitational wave observatory project
- Pre-DECIGO: test version of DECIGO
- Pre-DECIGO : $z \sim 10$ (30 Msun BH-BH)
 $\sim 10^5$ events/yr
- DECIGO can see Pop III BH-BHs
when Pop III stars were born!
(Nakamura, Ando, Kinugawa et al. 2006)



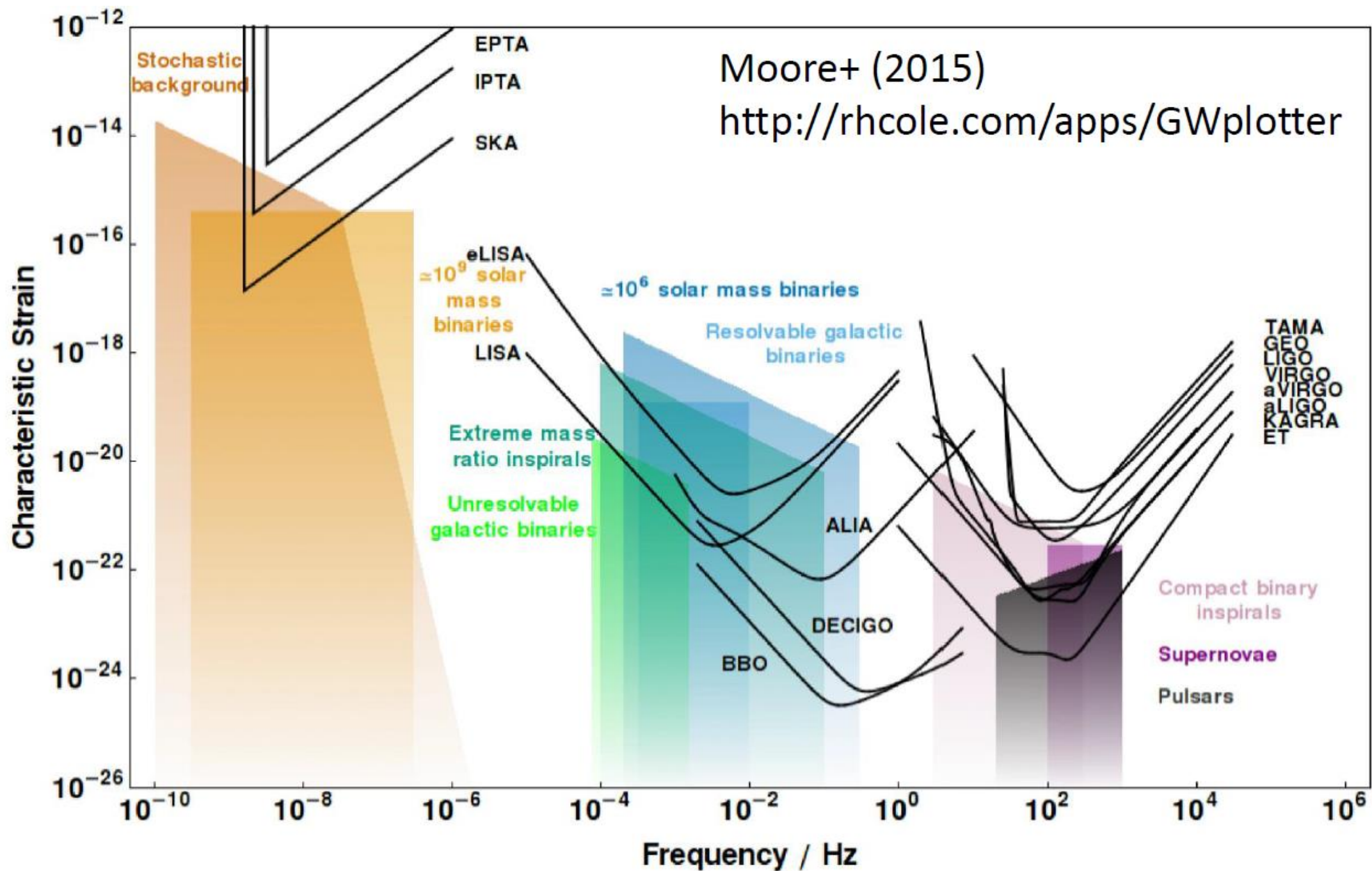
Summary

- Pop III binaries tend to become 30Msun+30Msun BH-BH
- **Pop III BBH detection rate of aLIGO in our standard model**
$$R \sim 180 \left(\frac{SFR_{peak}}{10^{-2.5}} \right) \left(\frac{f_b/(1+f_b)}{0.33} \right) [\text{yr}^{-1}] (S/N > 8)$$
- The mass distribution or the redshift dependence might distinguish Pop III from Pop I,II.
- DECIGO can see Pop III BH-BH merger when they were born

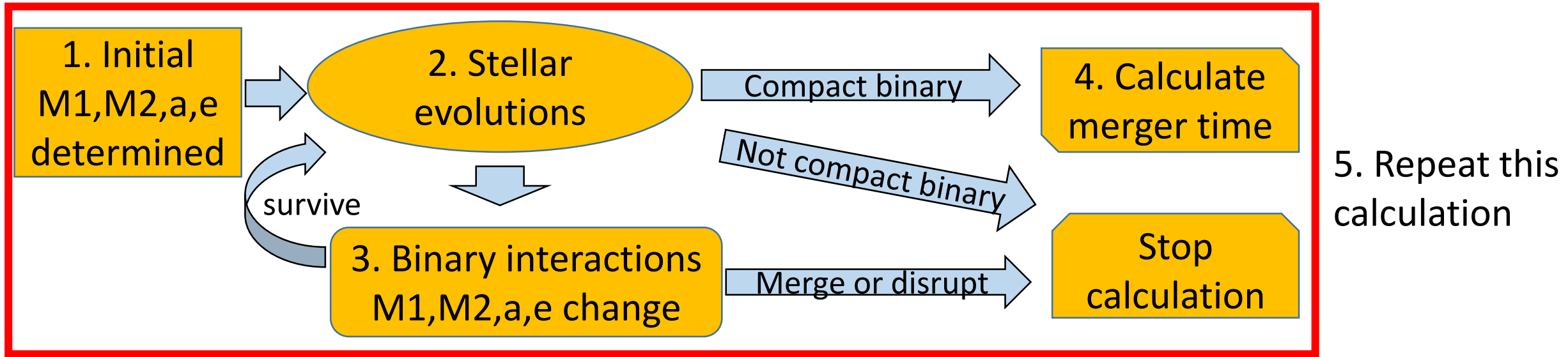
Appendix

Pop I and Pop II case (Dominik et al. 2015)

- From $1/200 Z_{\text{sun}}$ to $1.5 Z_{\text{sun}}$
- BH-BH detection rate (Their standard model) $\sim 300/\text{yr}$
- 25% of above rate is $>20 M_{\text{sun}}$ BHBH
- Thus, Detection rate of high mass BHBH $\sim 80/\text{yr}$



How to calculate Pop III binaries?

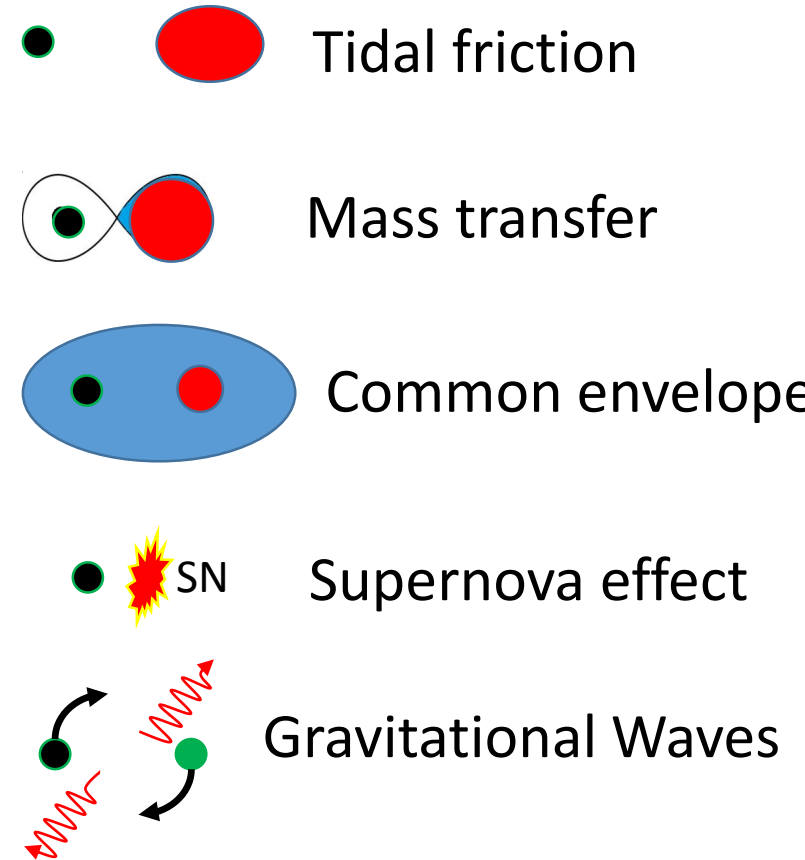


1. Initial stellar parameters are decided by Monte Carlo method with initial distribution functions (primary mass: M_1 , secondary mass: M_2 , separation: a , orbital eccentricity: e)
2. We calculate evolution of stars
3. If star fulfills the condition of binary interactions (BIs), we calculate BIs and change M_1 , M_2 , a , e .
 - If binary merges or disrupts due to BIs before binary becomes compact binary, we stop calculation.
 - If binary survives from BIs, we calculate stellar evolutions again.
4. If binary becomes compact binary (NS-NS, NS-BH, BH-BH), we calculate when binary merge due to GW.
5. We repeat these calculations and take the statistics of compact binary mergers.

Binary Interactions

- Tidal friction
- Mass transfer
- Common envelope
- Supernova effect
- Gravitational radiation

Change
 M_1, M_2, a, e



We need to specify some parameters to calculate these effects.

We use the parameters adopted for Pop I population synthesis in Our standard model.

Pop III binary population synthesis

We simulate 10^6 Pop III-binary evolutions and estimate how many binaries become compact binary which merges within Hubble time.

× 84 models (Kinugawa et al.2016)

Initial stellar parameters are decided by Monte Carlo method with initial distribution functions

- Initial parameter (M_1, M_2, a, e) distribution in our standard model

M_1 : Flat ($10 M_{\odot} < M < 100 M_{\odot}$)

$q = M_2/M_1$: $P(q) = \text{const.}$ ($0 < q < 1$)

a : $P(a) \propto 1/a$ ($a_{\min} < a < 10^6 R_{\odot}$)

e : $P(e) \propto e$ ($0 < e < 1$)



The same distribution functions
adopted for Pop I population
synthesis

Results

The numbers of the compact binaries which merge within Hubble time for 10^6 binaries

NSNS	NSBH	BHBH
0	50	115056

Our standard model

- A lot of Pop III BH-BH binaries form and merge within Hubble time
- Close NS binaries do not form

The star formation rate of Pop III

In order to calculate merger rate,
we need to know

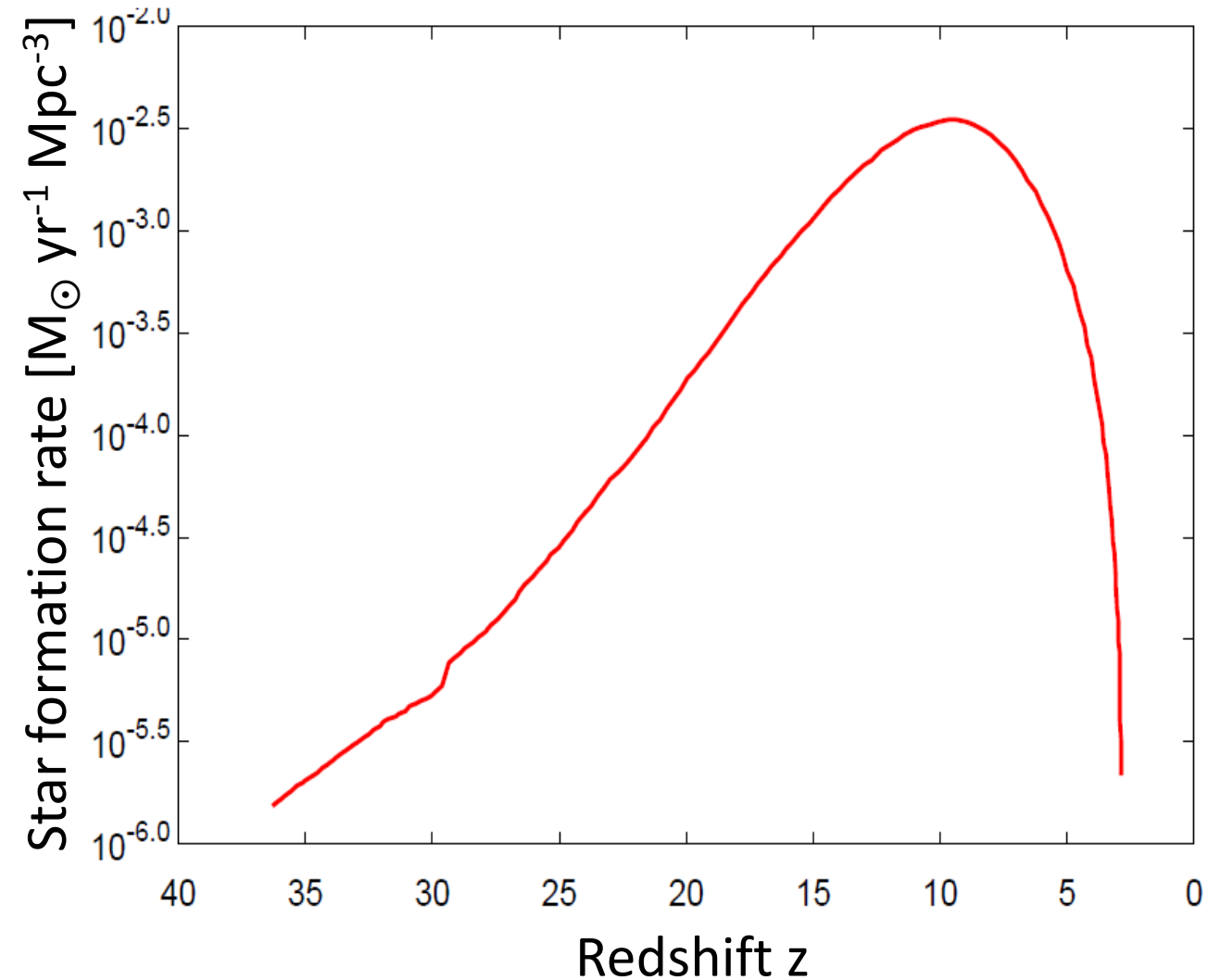
- When were Pop III stars born?
- How many Pop III stars were born?

⇒ Star formation rate

We adopt the Pop III SFR

by de Souza et al. 2011

$$SFR_{peak} \sim 10^{-2.5} [M_{\odot} \text{ yr}^{-1} \text{ Mpc}^{-3}]$$



(de Souza et al. 2011)

Consistency with LIGOS6 and Adv.LIGO

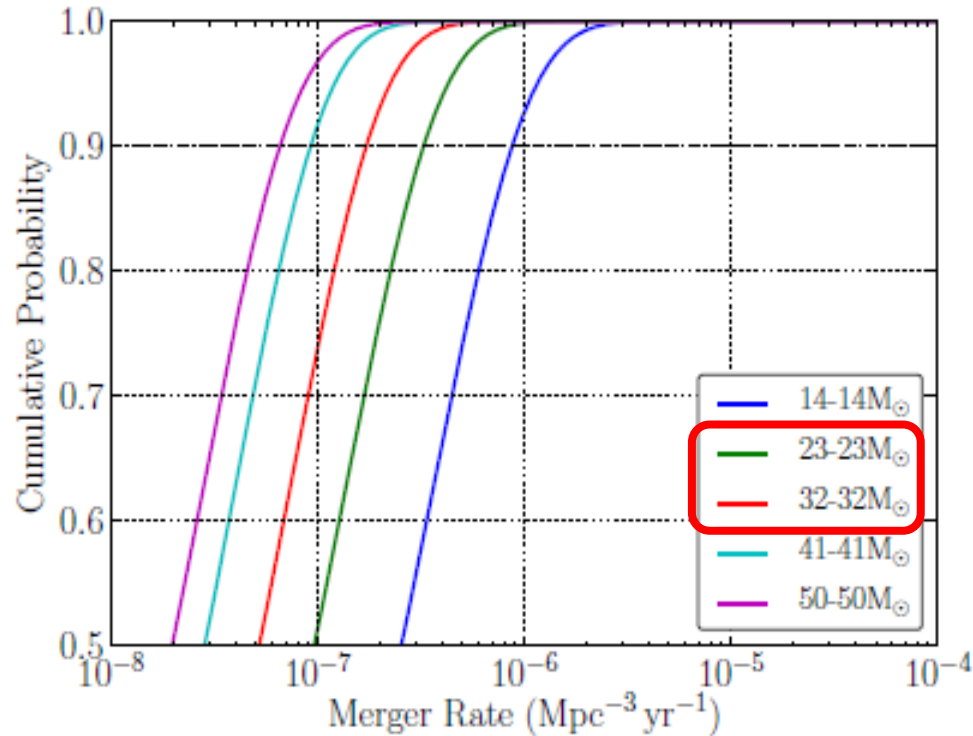


FIG. 6: Cumulative posterior probabilities over astrophysical merger rate, for the bins shown in Figure 5 with central values $m_1 = m_2 = 50, 41, 32, 23, 14 \text{ M}_\odot$ (left to right). We show the probability level corresponding to the 90% confidence rate limit (dashed horizontal line). These posteriors were evaluated for signals described by the EOBNRv2 waveform family in S6 data using S5 search results as prior information.

Aasi, Abadie, Abbott et al. (2013)

- **LIGOS6 upper limit of BH-BH merger rate**
left figure
 $\sim 10^{-7} \text{ yr}^{-1} \text{ Mpc}^{-3}$
- **Merger rate estimated by GW150914 ($z < 0.5$)**
 $\sim 0.02\text{-}4 \times 10^{-7} \text{ yr}^{-1} \text{ Mpc}^{-3}$
- **Pop III BH-BH Merger rate at $z \sim 0$**

$$R \sim 2.5 \times 10^{-8} \left(\frac{SFR_{peak}}{10^{-2.5}} \right) \text{Err}_{\text{sys}} [\text{yr}^{-1} \text{ Mpc}^{-3}]$$

Our result is consistent with LIGO

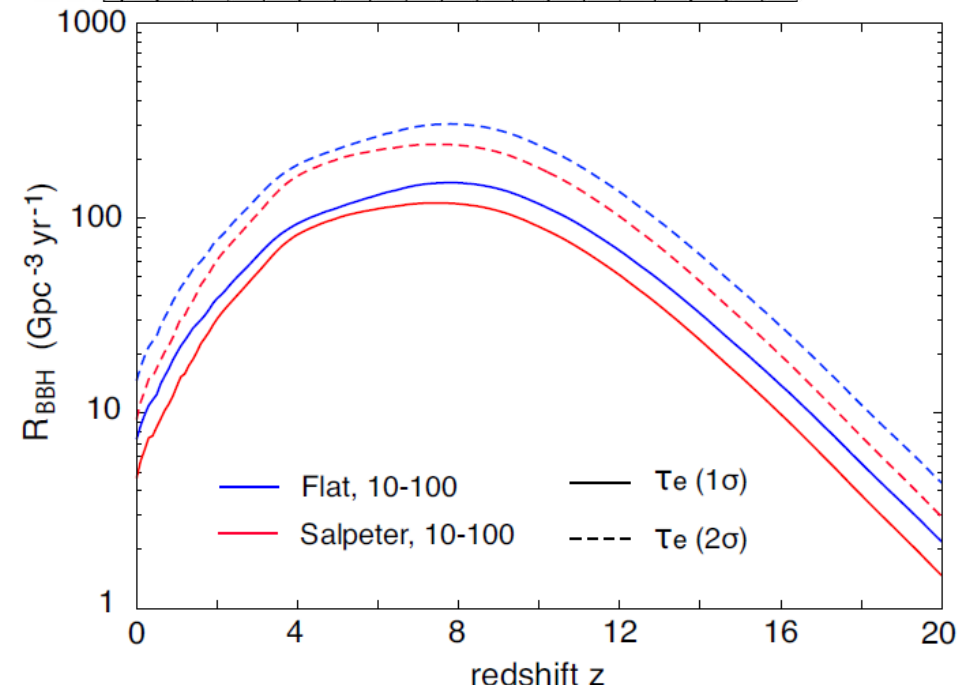
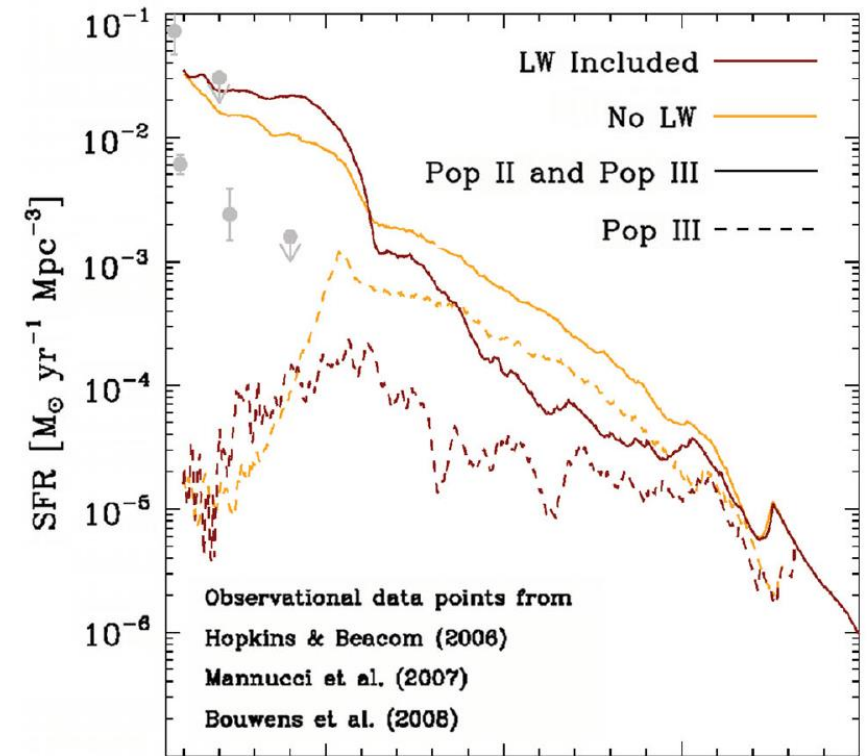
Errsys (Example)

	Errsys
Standard	1 (180 /yr)
Mass range: ($10 M_{\odot} < M < 100 M_{\odot}$ or $140 M_{\odot}$)	1~3.4
IMF: Flat, M^{-1} , Salpeter	0.42~1
IEF: $f(e) \propto e$, const., $e^{-0.5}$	0.94~1
BH natal kick: $V=0, 100, 300$ km/s	0.2~1
CE: $\alpha\lambda=0.01, 0.1, 1, 10$	0.21~1
Mass transfer (mass loss fraction): $\beta=0, 0.5, 1$	0.67~1.3
Worst	0.046

- On the other hand, the typical mass is not changed ($\sim 30 M_{\text{sun}}$).

Other Pop III SFRs

- SPH simulation
(Johnson et al. 2013)
 $\text{SFR} \sim 10^{-3} - 10^{-4} \text{ Msun/yr/Mpc}^3$
- Constraints by Planck
(e.g. Hartwig et al. 2016, Inayoshi et al. 2016)
optical depth of Thomson scattering
total Pop III density $\lesssim 10^{4-5} \text{ Msun/Mpc}^3$
by Visbal et al. 2015



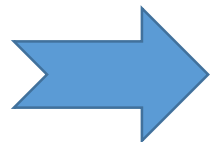
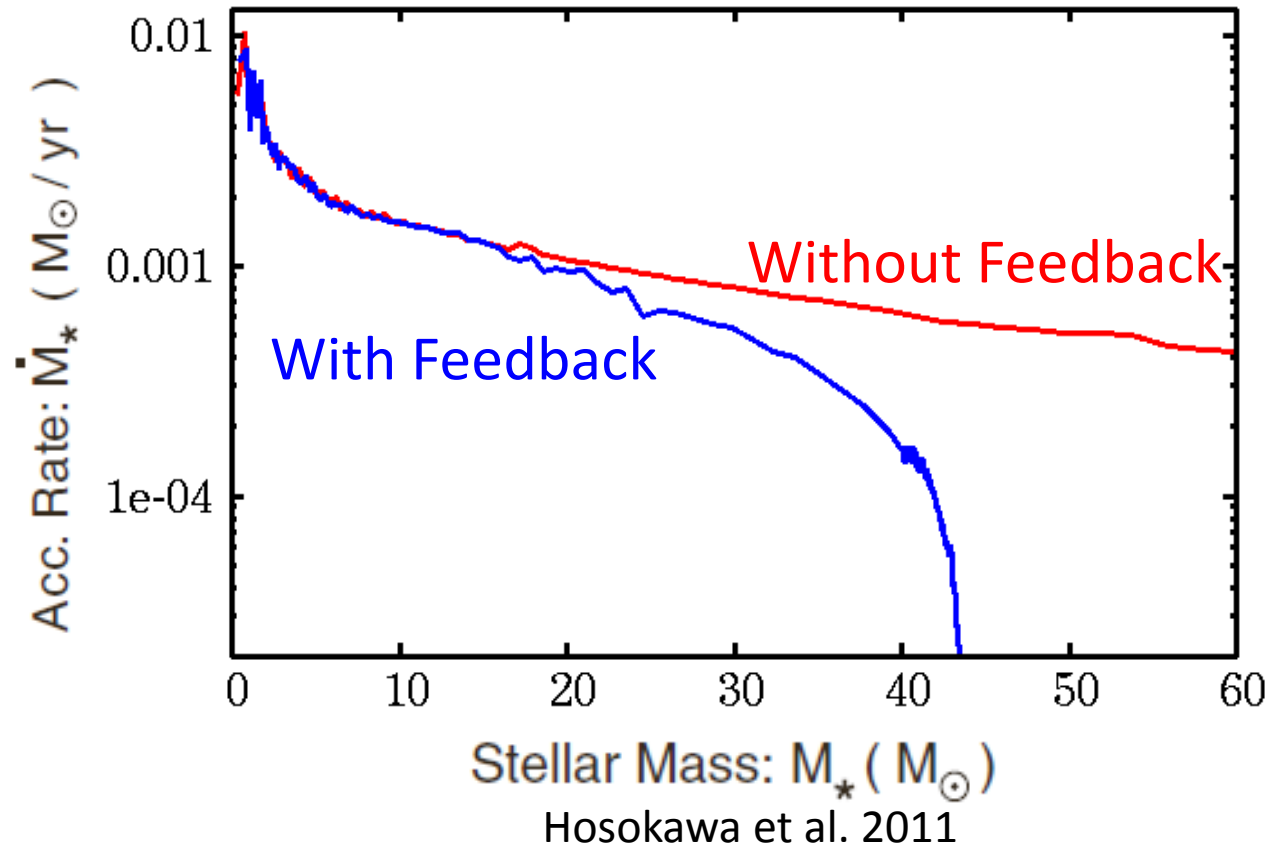
What is the expected Mass of Pop III stars ?

- Without UV feedback

The typical mass about $10^3 M_{\odot}$
(Omukai & Palla 2003, etc.)

- With UV feedback

The typical mass **10-100 M_{\odot}**
(Hosokawa et al. 2011, 2012)



Pop III stars \rightarrow 10-100 M_{\odot}

The differences between Pop III and Pop I

	Pop I stars (Sun like stars)	Pop III stars
Metallicity	2%	0
Radius	Large	Small
Typical Mass	1 Msun	10-100 Msun
Wind mass loss	effective	Not effective

Pop III binaries are easier to be massive compact binary

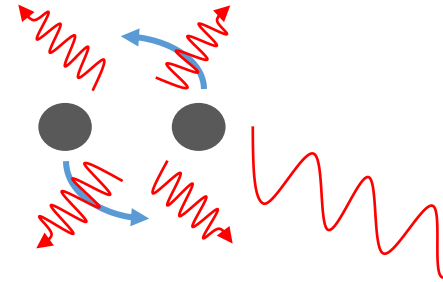
The main target of gravitational wave source

- Compact binary mergers

Binary neutron star (NS-NS)

Neutron star black hole binary (NS-BH)

Binary black hole (BH-BH)

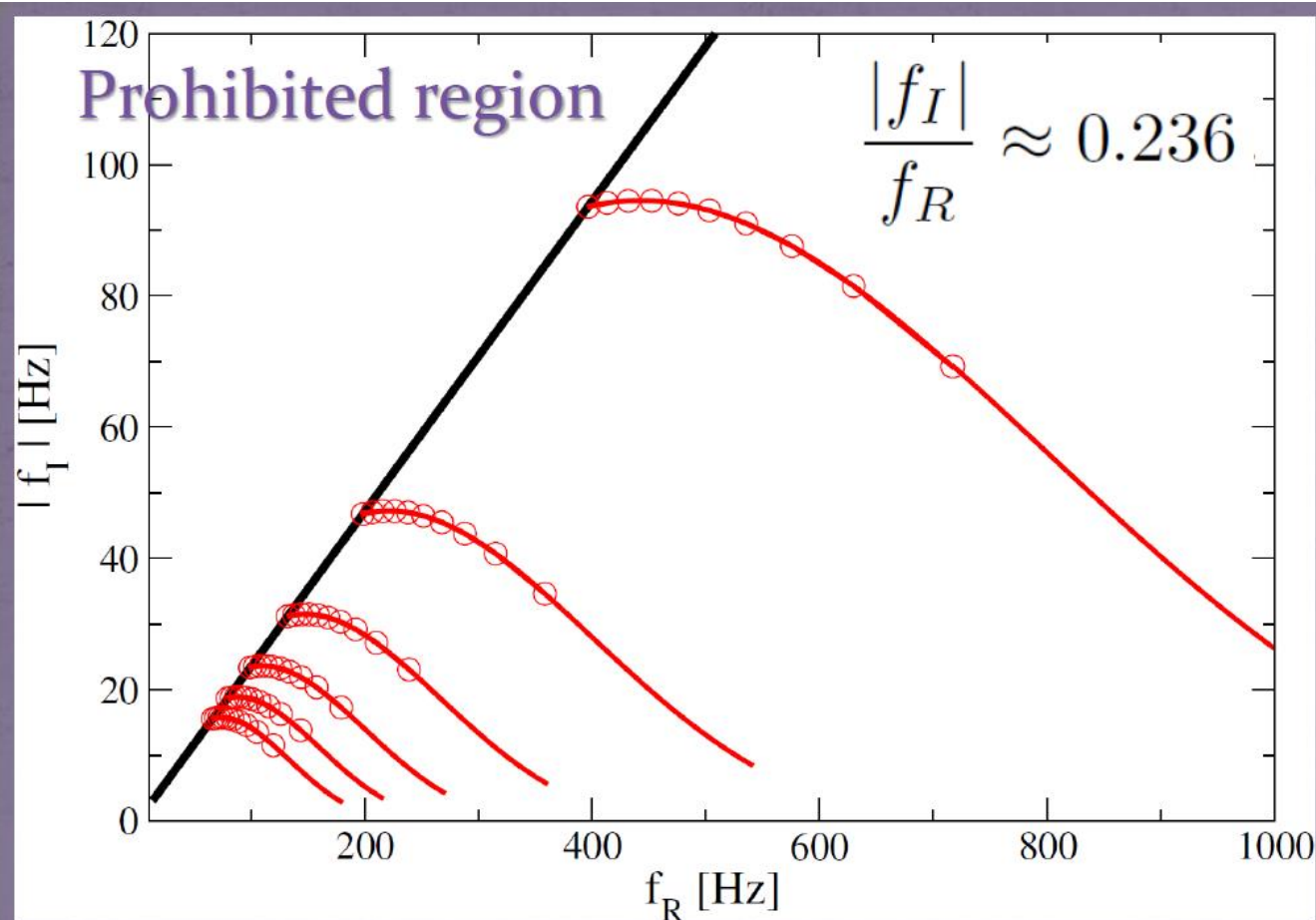


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How many times can we detect compact binary mergers ?

→ Estimated by the binary population synthesis

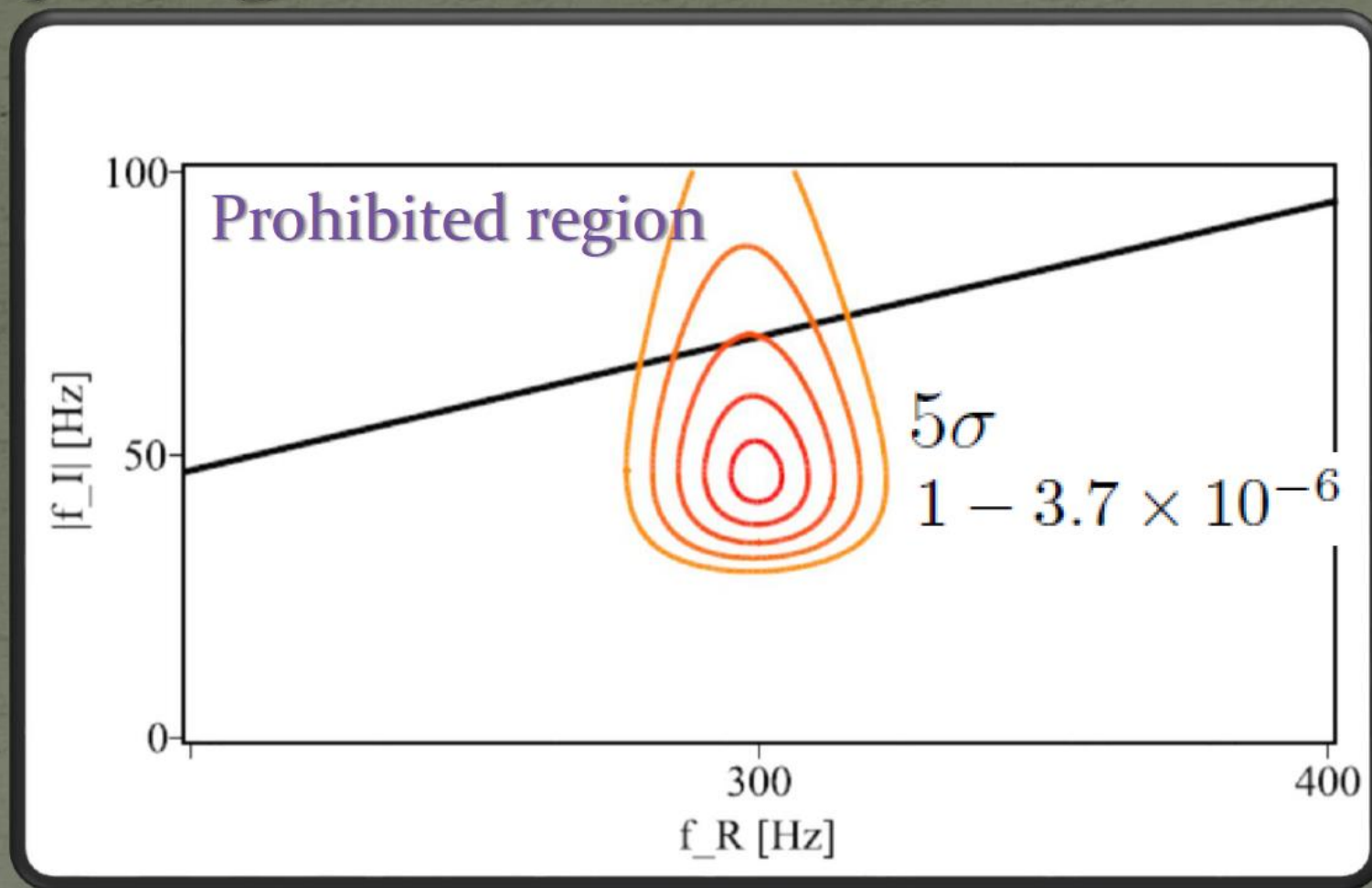
Quasi normal mode



- f_c is frequency of QNM
- Q is the quality factor of QNM which relate to the attenuation of QNM

$$f_R = f_c, \quad f_I = -\frac{f_c}{2Q},$$

Only ringdown



- SNR = 20 (500Mpc) for the typical Pop III BBH

$$(M = 60M_{\odot}, \eta = 1/4) \rightarrow (M_{\text{rem}} = 57.09M_{\odot}, \alpha_{\text{rem}} = 0.6867)$$
$$f_R = 299.5\text{Hz}, f_I = -46.34\text{Hz}, (f_c = 299.5\text{Hz}, Q = 3.232)$$

How to calculate the event rate

- NS-NS

We can get information from binary pulsar observations

- The empirical rate from pulsar observations (Kalogera et al. 2004,etc)
- Binary population synthesis(Belczynski et al. 2002, 2004, Dominik et al.2012,etc)

- NS-BH,BH-BH

- Binary population synthesis

There were no observation until GW150914.

Thus, there is no other way except binary population synthesis

Why do Pop III stars have these properties?

- **Zero metal** stars

- No line cooling and dust cooling at the star formation

- High temperature and high Jeans mass ($M_J \propto T^{3/2}$)

- ⇒ **More massive** than Pop I stars (Pop I stars are solar like stars)

- The typical mass is **10-100M_⊙**

- Missing metal and dust i.e. missing powerful opacity source

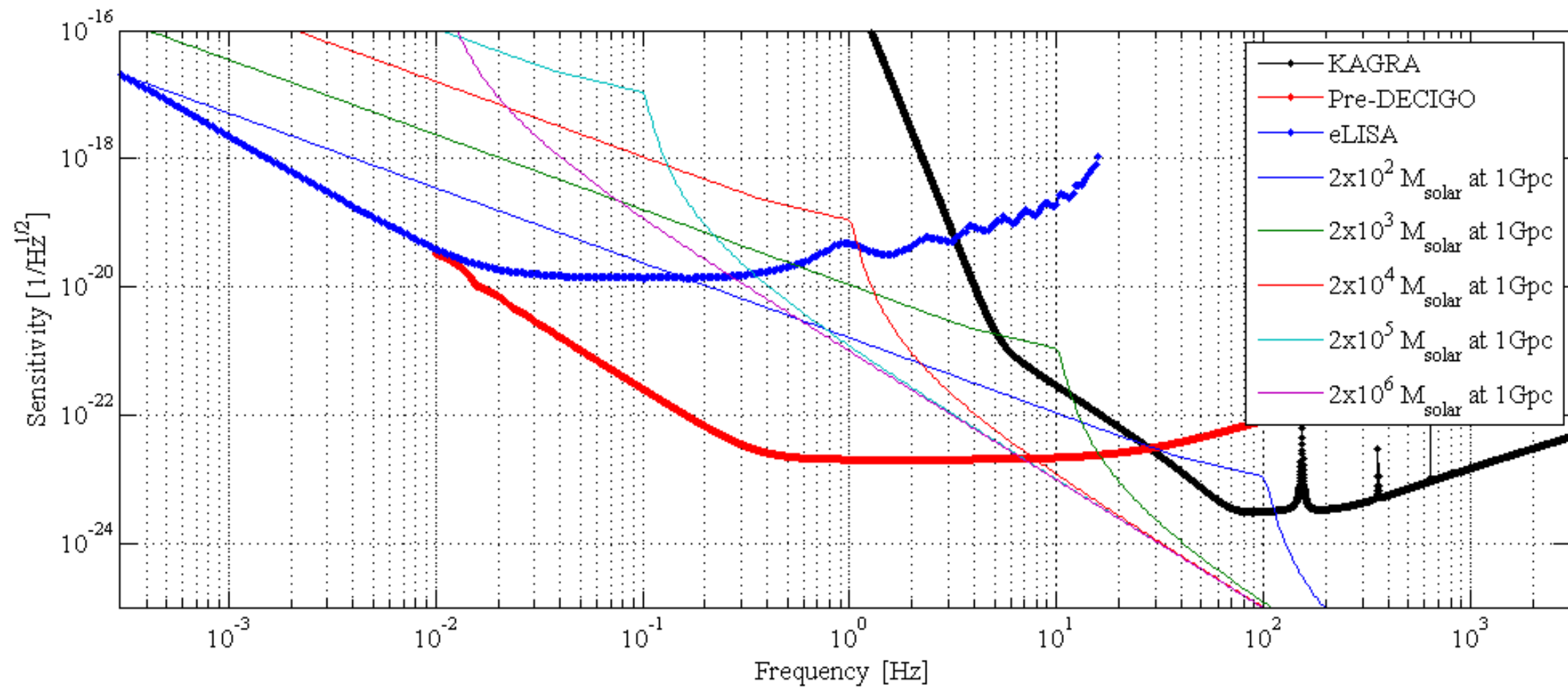
- The stellar photosphere become small

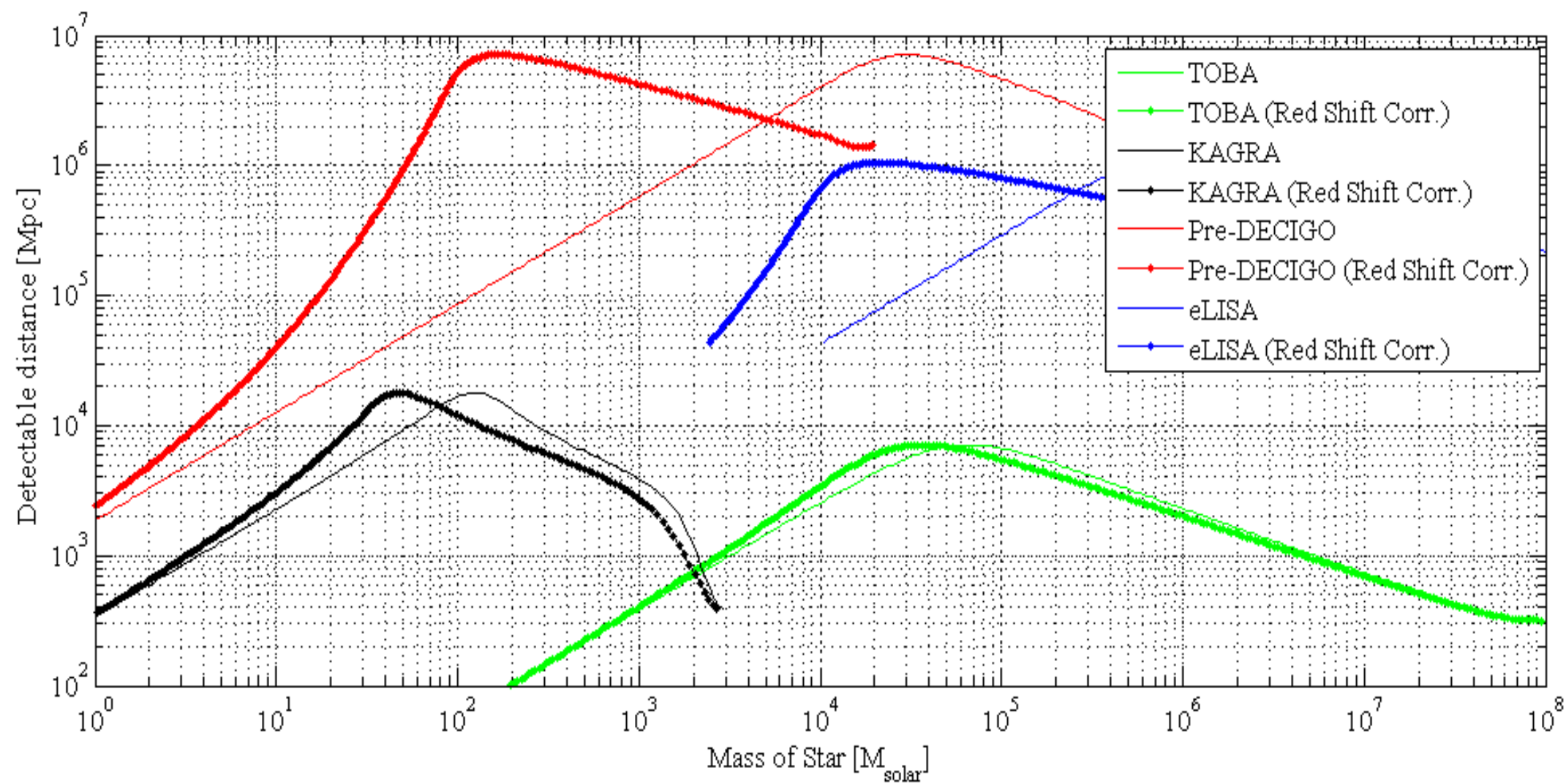
- ⇒ **Smaller radius** than Pop I stars

- Stellar wind is driven by radiation pressure on resonance lines of heavier ions or dust grains

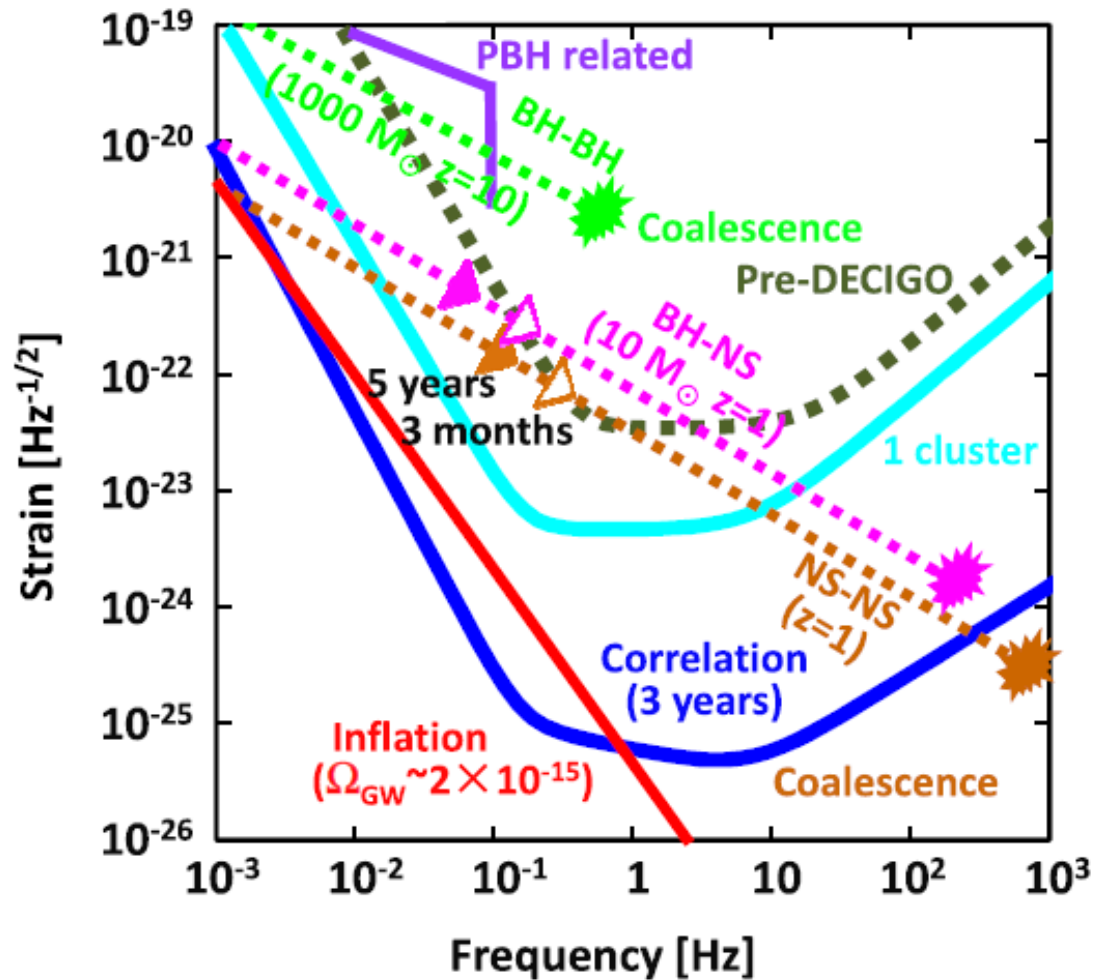
- However, Pop III stars do not have heavier ion and dust grain

- ⇒ **No wind mass loss**





DECIGOの感度曲線



Kawamura et al. 2011

- Pop III のSFRのピークは $z \sim 9$
- Red shift chirp mass = $(1+z)Mc$
- Pop III BHBH ($z \sim 9$) \Rightarrow 300 M_{sun} (10Hz)

How to calculate the event rate

- NS-NS

We can get information from binary pulsar observations

- The empirical rate from pulsar observations (Kalogera et al. 2004,etc)
- Binary population synthesis(Belczynski et al. 2002, 2004, Dominik et al.2012,etc)

- NS-BH,BH-BH

- Binary population synthesis

There is no observation.

Thus, there is no other way except binary population synthesis

merger rate calculated by population synthesis

Pop I galactic merger rate [Myr^{-1}] Dominik et al.(2012)

Model	NS-NS	BH-NS	BH-BH
S	23.5 (7.6)	1.6 (0.2)	8.2 (1.9)
V1	0.4 (0.4)	0.002 (0.002)	1.1 (1.1)
V2	11.8 (1.1)	2.4 (0.08)	15.3 (0.4)
V3	48.8 (14.3)	4.6 (0.03)	5.0 (0.03)
V4	20.8 (0.3)	0.9 (0.0)	0.3 (0.0)
		•	
		•	
		•	
V15	39.8 (17.8)	0.01 (0.007)	1.1 (1.0)
Range	0.4–77.4 (0.3–17.8)	0.002–10.6 (0.0–3.9)	0.05–29.7 (0.0–4.2)

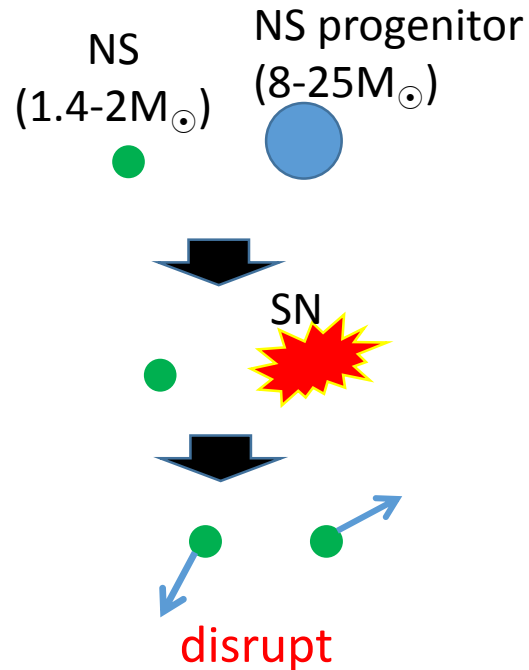
These merger rates are calculated by **Population synthesis (PS)**.

There are wide differences between models.

I will talk about what is PS and what determine the merger rates.

Why NS-NS disrupt

For example, we consider NS and NS progenitor binary.



In the case of Pop III NS progenitor, wind mass loss and the mass loss due to binary interaction is not effective. When NS progenitor becomes supernova, NS progenitor suddenly loses mass and becomes NS.

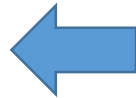
Then, due to instant mass loss the binding energy of binary decreases and binary NS disrupts.

➡ Binary NS **cannot survive!**

Binary Interactions

- Supernova effect
- Common envelope
- Stable mass transfer
- Orbital evolution

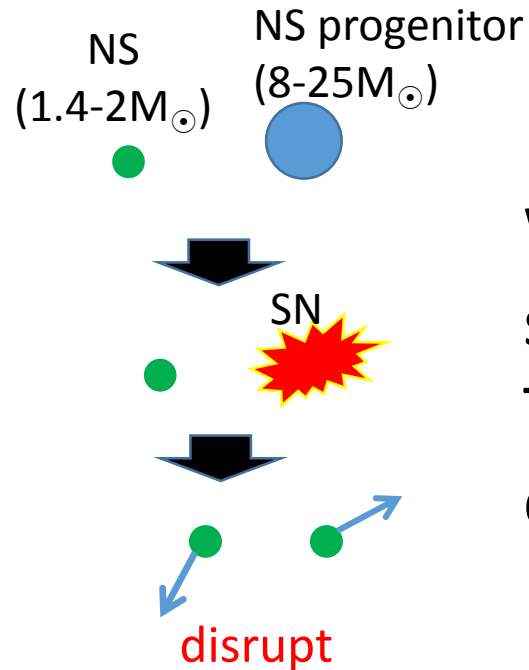
(Tidal friction, Gravitational radiation)



In this talk, I will explain these two binary interactions.

Supernova(SN) effect

For example, we consider NS and NS progenitor binary.



When NS progenitor becomes supernova, NS progenitor suddenly loses mass and becomes NS.

Then, due to instant mass loss the binding energy of binary decreases and binary NS disrupts.

➡ Binary NS **cannot survive!**

But in fact binary pulsars have been observed.

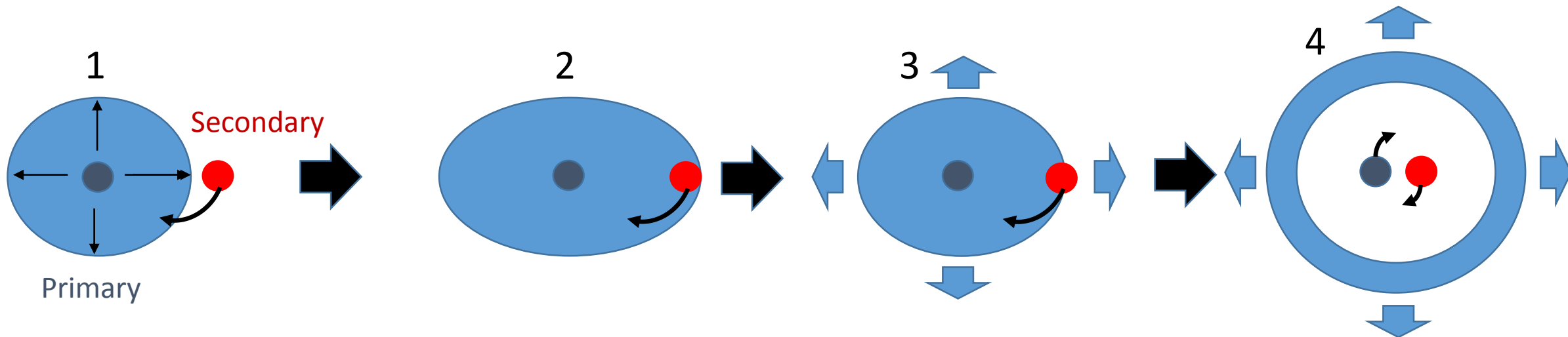
Why can binary NS survive?

This reason is **common envelope**.

Common envelope (CE)

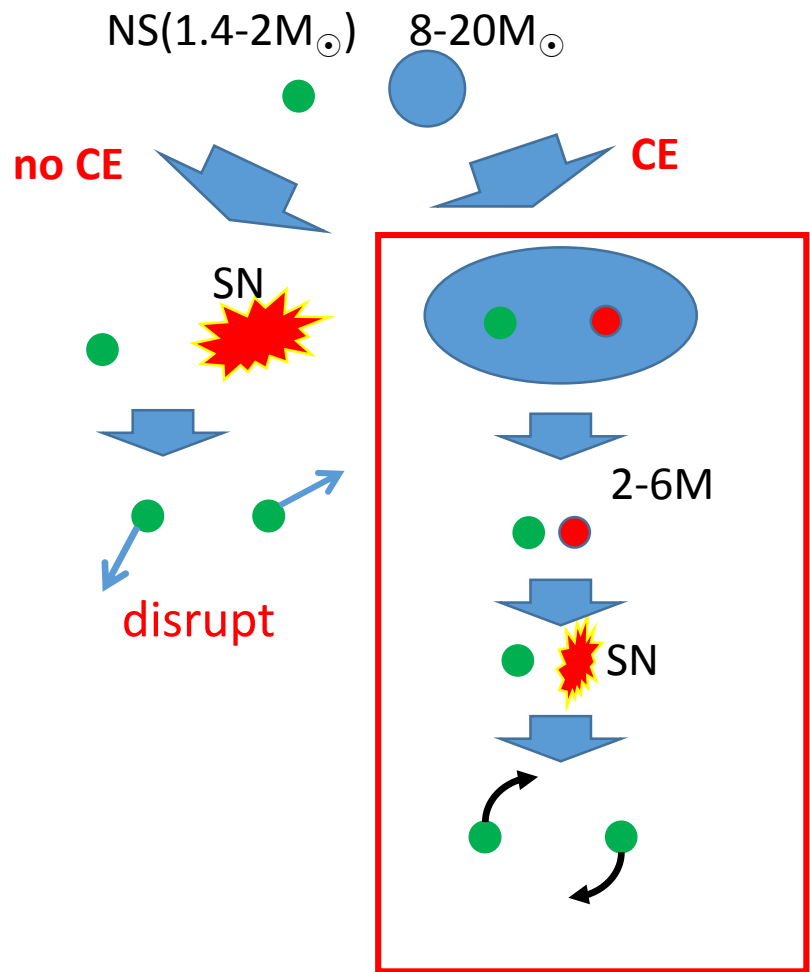
CE is unstable mass transfer phase.

1. Primary star becomes giant and primary radius becomes large.
2. Secondary star plunges in primary envelope.
3. The friction occurs between secondary and primary envelope and transfers angular momentum and energy from orbit to envelope. Due to orbital energy transfer **separation decreases** and **envelope expands and will be expelled**.
4. Binary becomes **close binary or merges** during CE.



Can NS binary survive via CE?

We consider NS and NS progenitor binary again.



If CE occurs, envelope was already expelled before SN. Thus, mass ejection at SN becomes smaller than SN mass ejection via no CE.

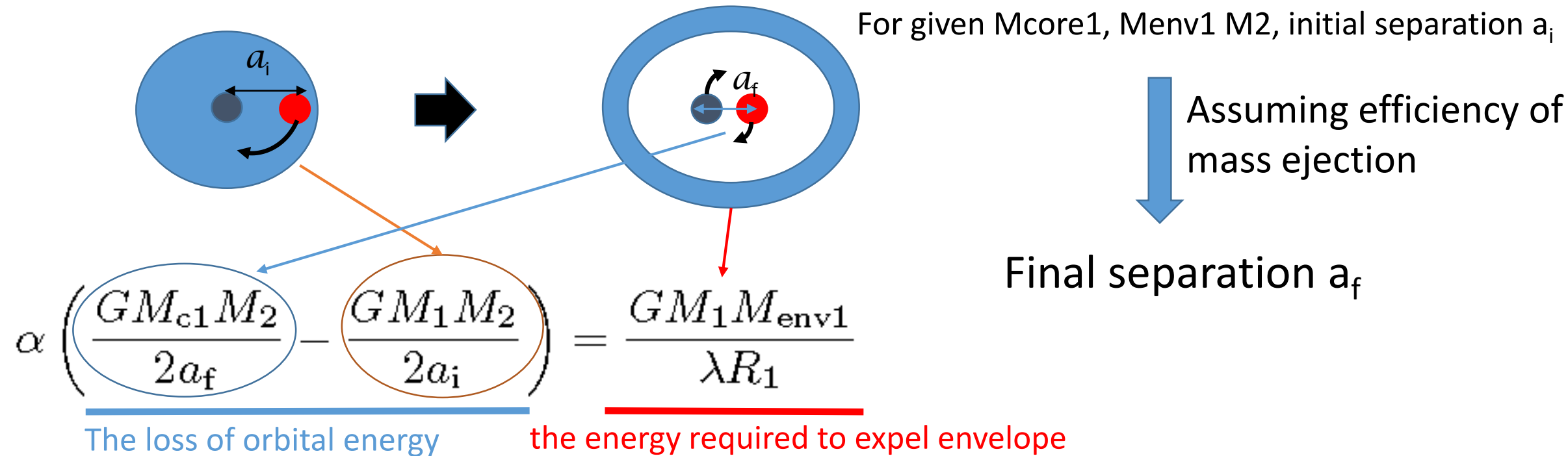
Due to small mass ejection at SN the loss of binding energy becomes small.

Binary can **survive** !

Therefore, **Common Envelope** is important.

The treatment of CE

- We assume the fraction of the orbital energy is used to expel envelope.
- We use simple energy formalism in order to calculate separation after CE a_f



α : the efficiency of energy transfer from orbit to envelope

λ : the binding energy parameter

These common envelope parameters are **uncertain**.

- How much the orbital energy can be used to expel envelope?
- How much the internal energy of envelope is used to expel envelope?

The rate dependence on CE parameters

$$\alpha \left(\frac{GM_{c1}M_2}{2a_f} - \frac{GM_1M_2}{2a_i} \right) = \frac{GM_1M_{env1}}{\lambda R_1}$$

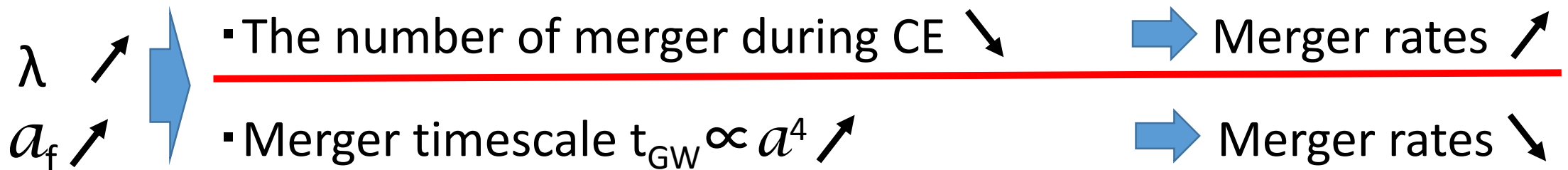
The loss of orbital energy
the energy required to expel envelope

- Separation after CE a_f is dependent on CE parameters.

For simplicity, $\alpha=1$.

If λ is large i.e, the energy required to expel envelope is small, the loss of orbital energy during CE becomes small and a_f is large.


- If a_f is large, binary tend not to merge during CE and can survive.
- However, if a_f is too large, binary cannot merge within Hubble time due to GW.



The dependence on CE parameters

For example, we consider how Pop I NS-NS merger rate depend on CE parameters.

Pop I NSNS merger rate [$\text{Myr}^{-1} \text{ galaxy}^{-1}$] Dominik et al.2012

	parameter	NS-NS merger rate [events/yr/galaxy]	
$\alpha\lambda$ $a_f \nearrow$	$\alpha\lambda = 0.01$	0.4	
	$\alpha\lambda = 0.1$	11.8	
	$\alpha\lambda = 1$	48.8	
	$\alpha\lambda = 10$	20.8	

▪ The number of coalescence during CE \searrow  Merger rates \nearrow

▪ Merger timescale $t_{\text{GW}} \propto a^4 \nearrow$  Merger rates \searrow

Binary population synthesis

- Population synthesis is a method of numerical simulation to research the population of stars with a complex evolutions.
- Population synthesis can predict properties and merger rates of unobserved sources such as NS-BH, BH-BH
- The common envelope of the key process of population synthesis
- However, Common envelope parameters are uncertain.

This uncertainty change event rate by a factor of several hundreds.

We should reveal this uncertainty via comparison between result of population synthesis and observations such as GW and other observations and improve binary evolution theory

Example: CE dependence

We calculate $\alpha\lambda=0.01, 0.1, 1, 10$ cases $N_{\text{total}}=10^6$

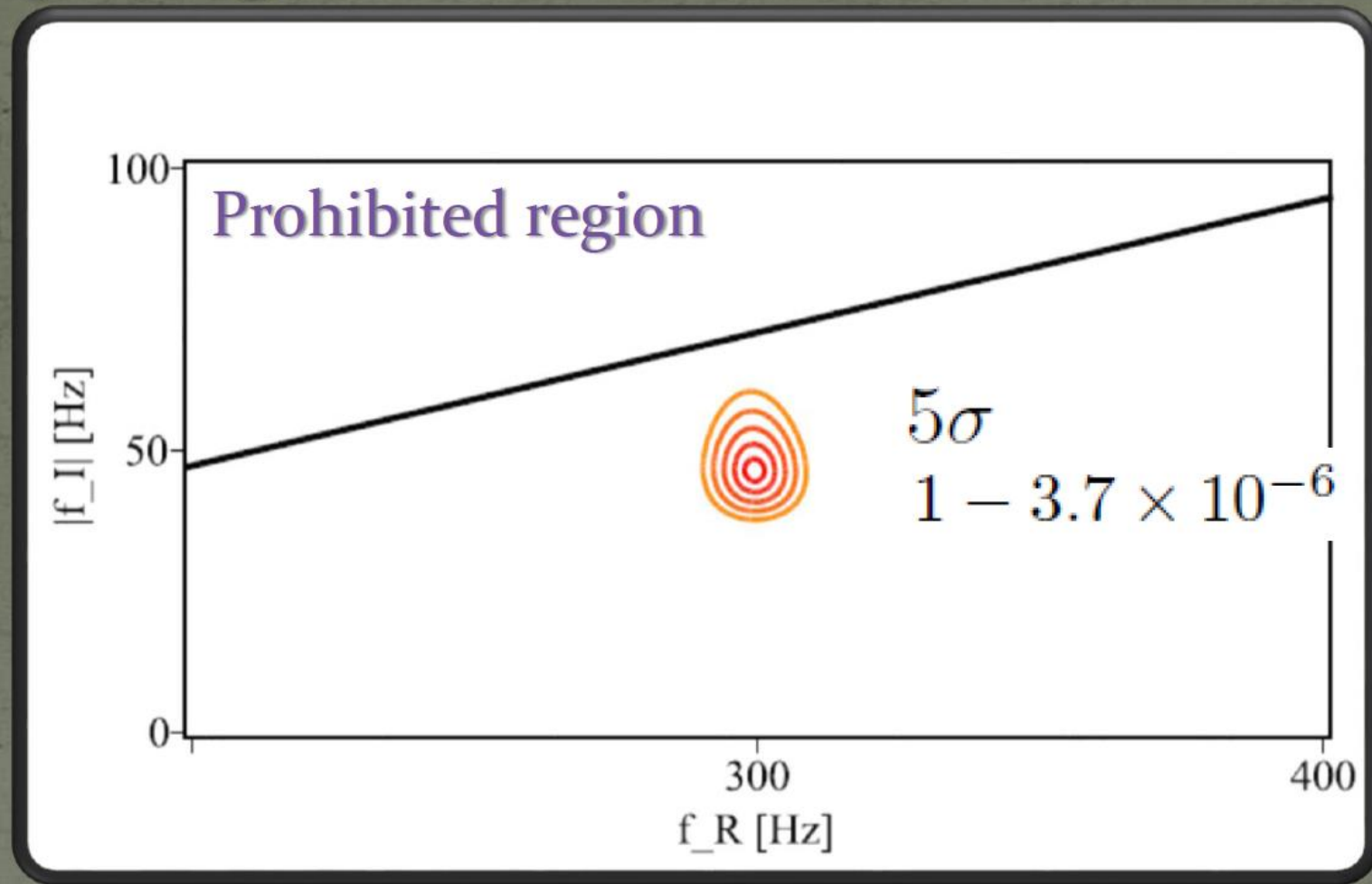
	Standard(1)	0.01	0.1	10
NSNS	0	0	0	1116
NSBH	185335	148290	162814	198408
BHBH	517067	340893	434590	542399
merged NSNS	0	0	0	890
merged NSBH	50	0	45	767
merged BHBH	115056	32283	111696	91787

The number of merged Pop III BH-BH change by a factor of *several*.

On the other hand, Pop I merger rates changed by a factor of *several hundreds*.

What is the reason?

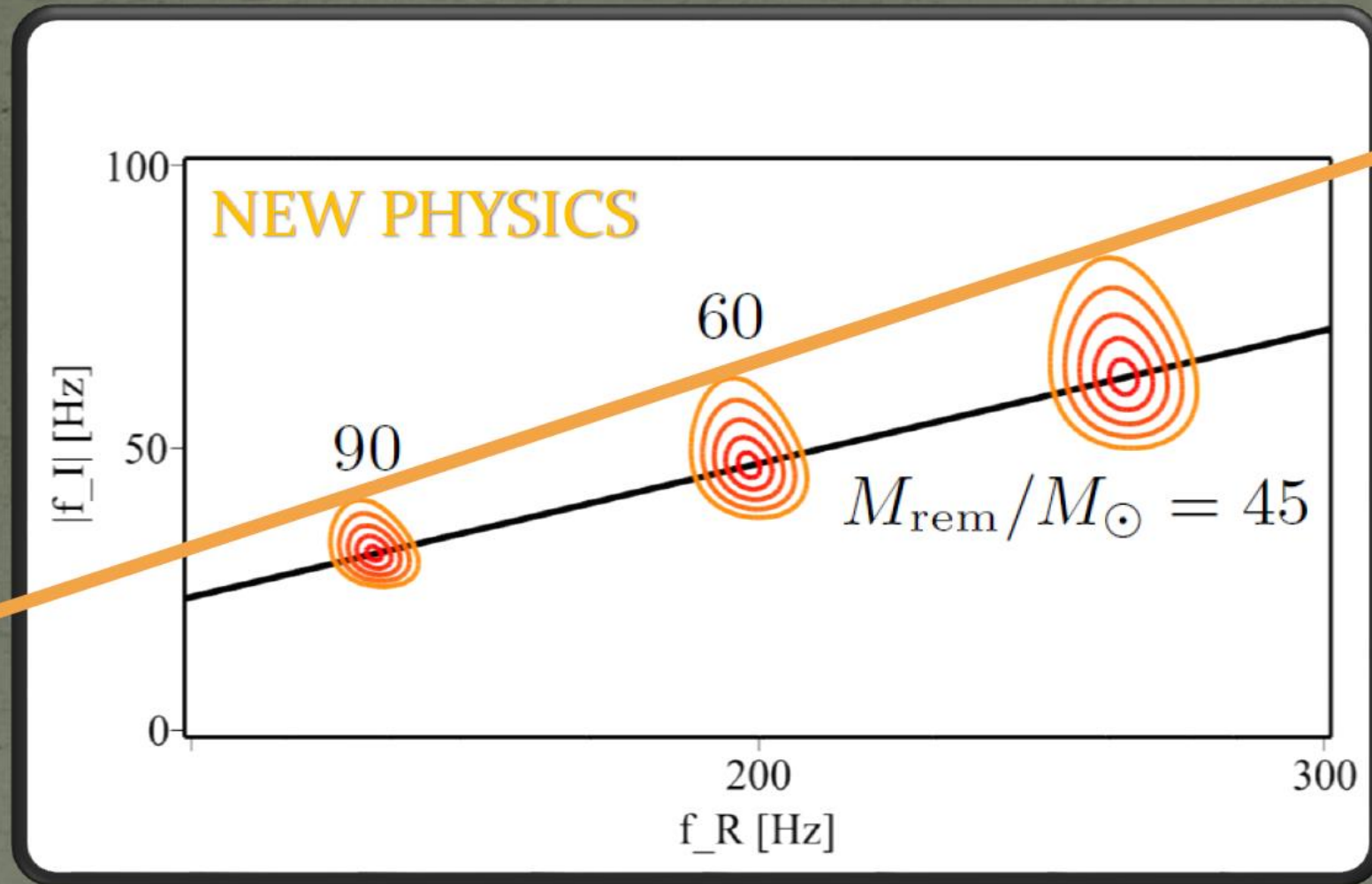
Only ringdown



- SNR = 50 (200Mpc) for the typical Pop III BBH

$$(M = 60M_{\odot}, \eta = 1/4) \rightarrow (M_{\text{rem}} = 57.09M_{\odot}, \alpha_{\text{rem}} = 0.6867)$$
$$f_R = 299.5\text{Hz}, f_I = -46.34\text{Hz}, (f_c = 299.5\text{Hz}, Q = 3.232)$$

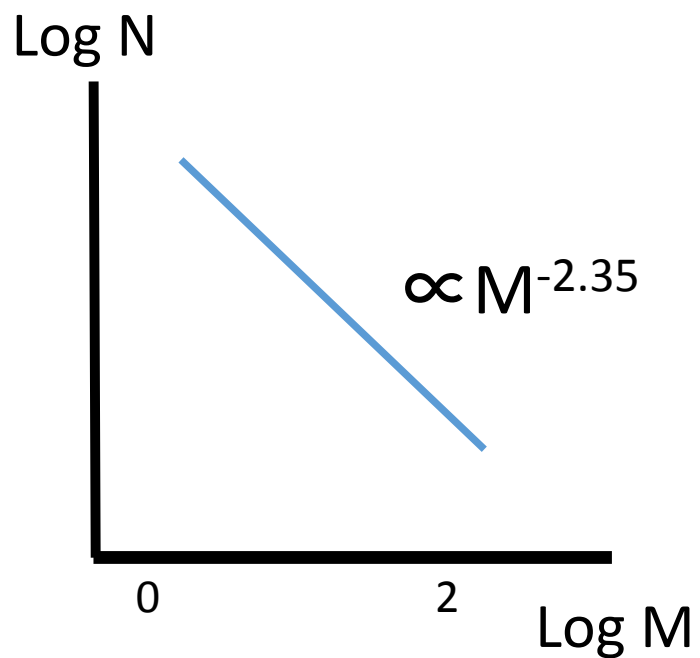
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- SNR = 50 (200Mpc), Schwarzschild case

IMF

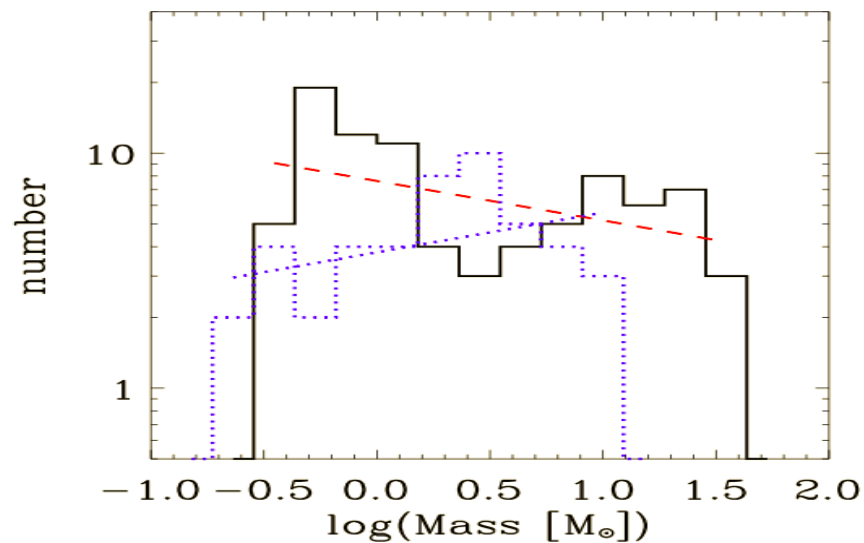
- Pop I
Salpeter



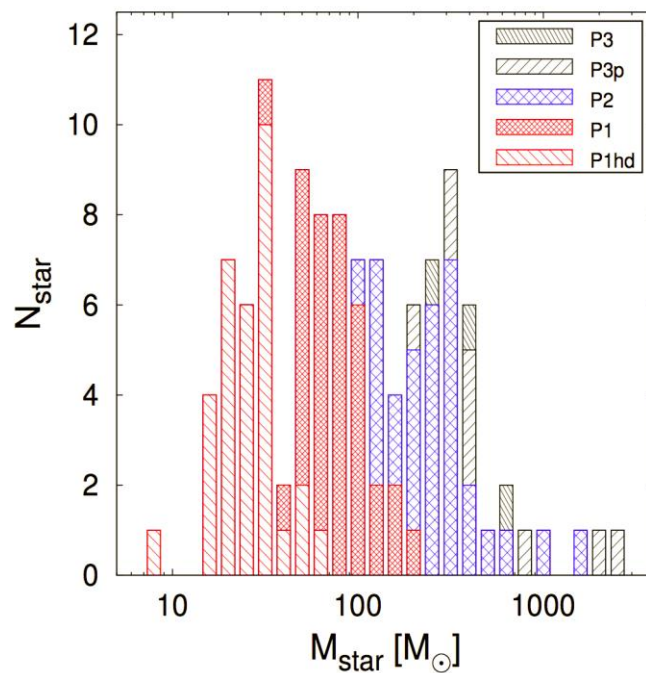
- Pop III

Flat ?

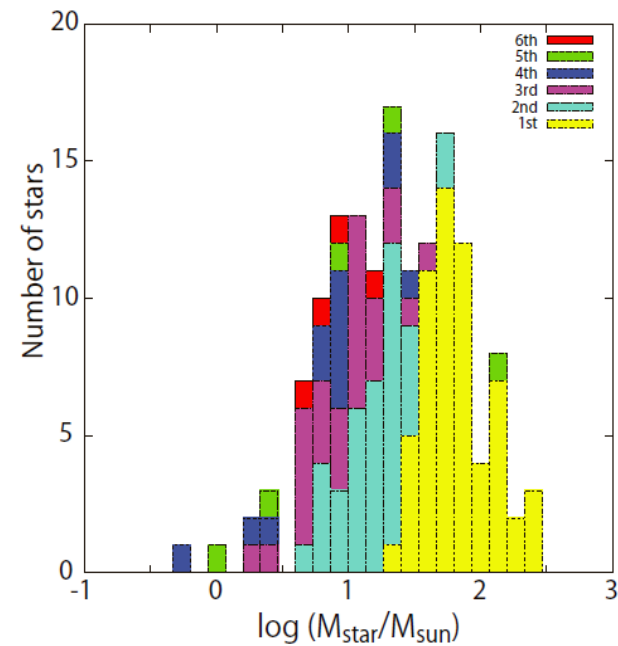
Log Flat ?



Stacy & Bromm 2013



Hirano et al.2014



Susa et al. 2014

IMF dependence

	Standard(Flat)	M^{-1}	Salpeter
NSNS	0	2	5
NSBH	185335	168100	93085
BHBH	517067	350169	132534
merged NSNS	0	2	5
merged NSBH	50	68	64
merged BHBH	115056	74745	25536

Uncertainties of Pop III binary population synthesis

- Initial condition

IMF

mass ratio

separation

eccentricity

- Binary interactions

Common envelope

Mass transfer

Supernova kick

eccentricity distributions

- General eccentricity distribution (Heggie 1975)
 $P(e) \propto e$ (Standard)
- Cygnus OB2 association (Kobulnicky et al. 2014)
 $P(e) = \text{const.}$
- Observations of O stars ($M > 15 M_{\text{sun}}$) (Sana et al. 2012)
 $P(e) \propto e^{-0.5}$

eccentricity dependence

	Standard(e)	const	$e^{-0.5}$
NSNS	0	0	0
NSBH	185335	183460	181650
BHBH	517067	522809	523285
merged NSNS	0	0	0
merged NSBH	50	43	38
merged BHBH	115056	111106	107594

Uncertainties of Pop III binary population synthesis

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Supernova kick

Mass transfer

$$\dot{M}_2 = -(1 - \beta)\dot{M}_1$$

- $\beta=0$: conservative
- $1>\beta>0$: non conservative

In Standard model, we use the fitting function

$$\begin{cases} \dot{M}_2 = \min\left(10\frac{\tau_{\dot{M}}}{\tau_{\text{KH},2}}, 1\right) \dot{M}_1 & \text{Secondary is MS or He-burning} \\ \dot{M}_2 = -\dot{M}_1 & \text{Secondary is giant} \end{cases}$$

(Hurley et al. 2002)

This is fitted for Pop I stars.

Thus, we check $\beta=0, 0.5, 1$ cases.

Mass transfer dependence

	Standard(func.)	0	0.5	1
NSNS	0	0	5	1359
NSBH	185335	185335	193921	218311
BHBH	517067	517067	549893	531452
merged NSNS	0	0	5	1358
merged NSBH	50	50	199	119
merged BHBH	115056	115056	117094	50119

Supernova kick

- Pulsar kick $\sim 200\text{-}500\text{km/s}$

Pulsar observation suggest NSs have the natal kick at the SN.

- BHXRBS have large distance from galactic plane.

Black hole natal kick? (Repetto, Davis & Sigurdsson 2012)

\Rightarrow We check the kick dependence.

$\sigma=0\text{km/s}$ (Standard)、 $\sigma=100\text{km/s}$ 、 $\sigma=300\text{km/s}$

$$P(v_k) = \sqrt{\frac{2}{\pi}} \frac{v_k^2}{\sigma_k^2} \exp \left[-\frac{v_k^2}{\sigma_k^2} \right]$$

SN kick dependence

	Standard(0)	100 km/s	300 km/s
NSNS	0	283	8
NSBH	185335	32701	11922
BHBH	517067	191755	70728
merged NSNS	0	17	1
merged NSBH	50	2527	3893
merged BHBH	115056	117415	51928