A Self-consistent Description of Black Hole Evaporation

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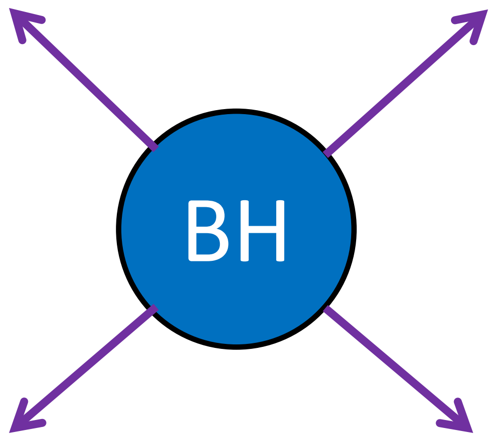
Motivation

- Black Holes evaporate.



$$T_H = \frac{\hbar}{4\pi a}$$

$$a \equiv 2GM$$

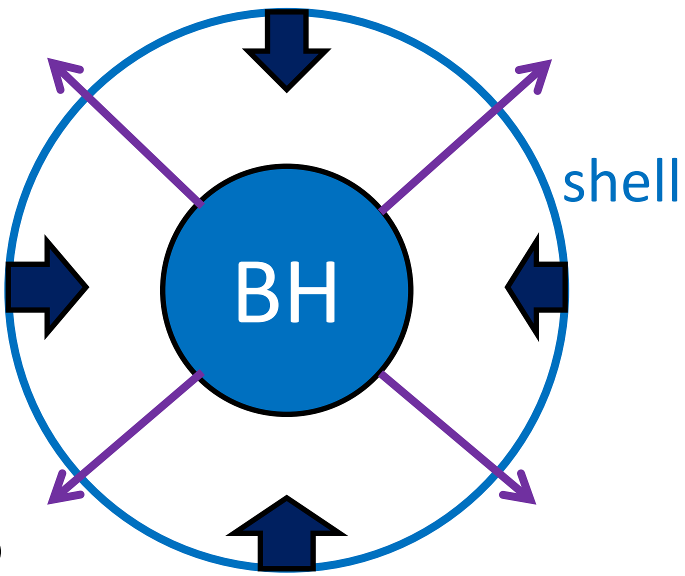
- We try to understand what this fact leads to.

⇒In the semi-classical approximation, we analyze time evolution of a collapsing matter, including the back reaction from evaporation.

Basic idea: step1

- Consider a spherically-symmetric evaporating BH in the vacuum.

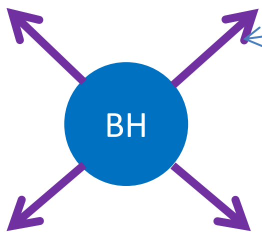
⇒Add a spherical thin shell.



⇒What happens?

⇒The shell will never reach the horizon.

Why will the shell never reach the horizon?



outside spacetime ≈ outgoing Vaidya metric

$$ds^2 = -\frac{r-a(u)}{r}du^2 - 2dudr + r^2 d\Omega^2$$

Stephan-Boltzmann law of $T_H = \frac{\hbar}{4\pi a}$: $$\frac{da(u)}{du} = -\frac{\sigma}{a(u)^2}$$ $(a \equiv 2GM)$

(intensity: $\sigma = O(1) \sim \hbar G \times N(\#fields)$)

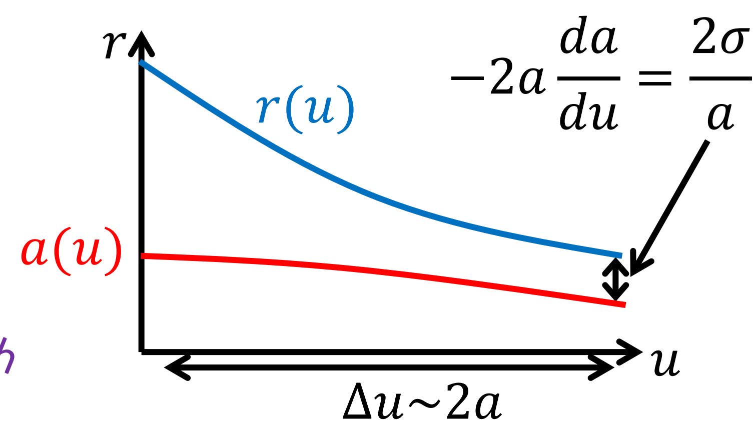
e.o.m. of the shell:

$$\frac{dr(u)}{du} = -\frac{r(u)-a(u)}{2r(u)}$$

$$\Rightarrow r(u) \approx a(u) - 2a(u)\frac{da(u)}{du} + Ce^{-\frac{u}{2a(u)}}$$

$$\longrightarrow a(u) + \frac{2\sigma}{a(u)} \equiv R(a(u))$$

$\sim\sqrt{N}l_p$ as proper length

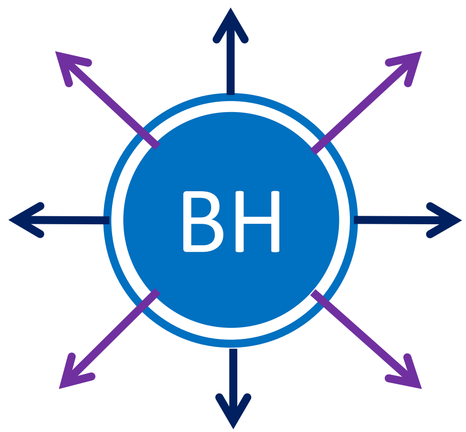


⇒The shell reaches not $a(u)$ but $R(a(u))$ because "$a(u)$ is escaping from $r(u)$."

Basic idea: step2

- After reaching $R(a')$, the shell starts to emit radiation.

$a' = 2G(m + \Delta\mathrm{m})$



$\Rightarrow$ (radiation from shell with Δm) $+$

(redshift) $\times$ (radiation from BH with m)

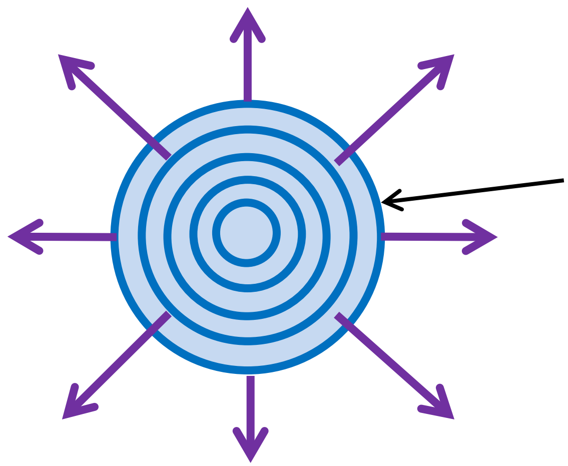
$=$ (radiation from BH with $m + \Delta m$)

$\Rightarrow$ The total system behaves like an ordinary BH with $m + \Delta m$, but it has no horizon at $r = 2G(m + \Delta m)$.

Basic idea: step3

- By using this procedure recursively, we can grow up a BH, which corresponds to formation of BH by a collapsing matter.

⇒We can consider the BH as consisting of many shells.



Surface at

$$r = R(a) = a + \frac{2\sigma}{a} \sim a + \frac{N l_p^2}{a} > a$$

⇒From outside, this object looks like an ordinary evaporating BH.

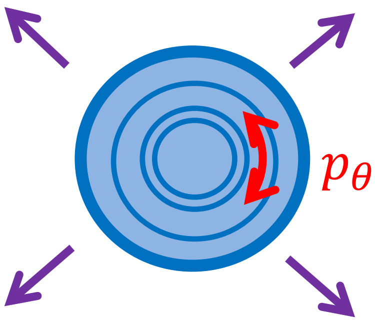
But it has an interior structure without trapped region or singularity.

⇒This object is the "BH"!

Why is this object stable?

- The 4d Weyl anomaly produces radiation and strong angular pressure p_θ, which supports the object:

TOV eq: $$\frac{dp_r}{dr} + \frac{2}{r}(p_r - p_\theta) + \frac{d\phi}{dr}(\rho + p_r) = 0 \Longrightarrow p_\theta \approx r\frac{d\phi}{dr}\rho$$



- The interior is not a fluid ($p_r \ll p_\theta$).

Note: In this sense, our discussion is very different from 2d models (e.g. CGHS).

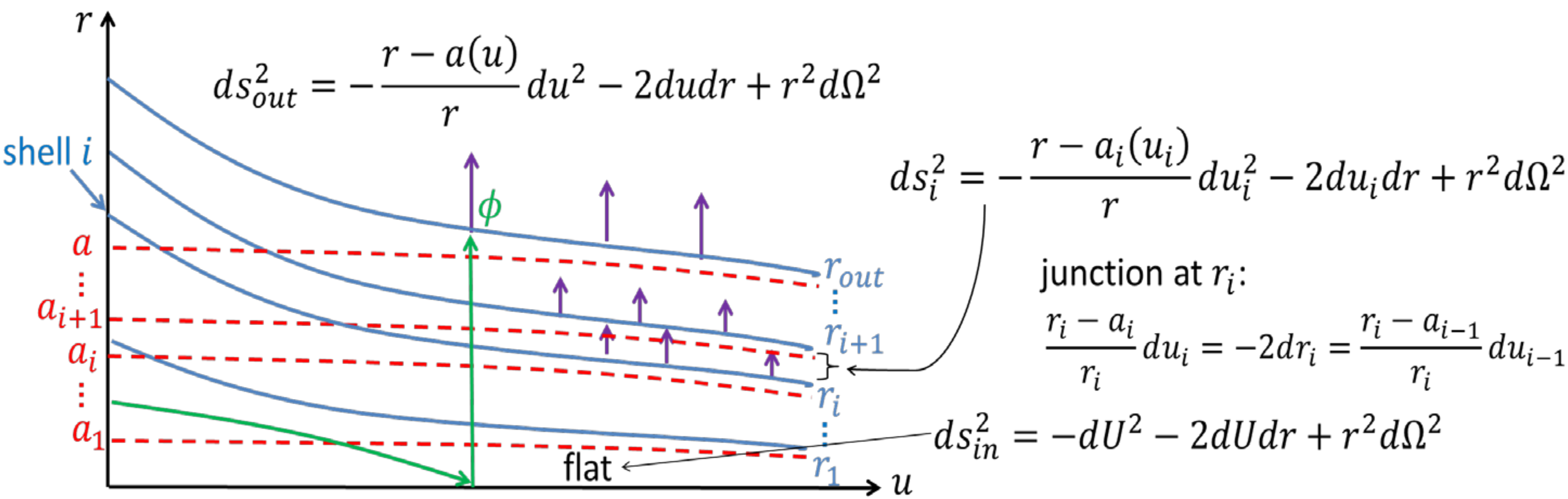
A self-consistent multi-shell model

We solve $G_{\mu\nu} = 8\pi G\langle T_{\mu\nu}\rangle$ and construct a self-consistent model that realizes the above picture.

Setup of the model

- Analyze time evolution of many classical thin null shells, including the back reaction from evaporation.

- Consider quantum massless scalar fields ϕ_a $(a = 1\sim N \gg 1)$, by taking only s-wave and using eikonal approximation.



Evaluation of $G_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle$ and derivation of Hawking radiation

Model (2/3)

- We can solve the Heisenberg eq $\nabla^2 \phi = 0$ and construct operator $T_{\mu\nu}$ by point-splitting regularization.

$\Rightarrow G_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle$ and $\nabla^2 \phi = 0$ reduce to

$$\frac{da_i}{du_i} = -\frac{N l_p^2}{8\pi}\left[\frac{\ddot{U}(u_i)^2}{\dot{U}(u_i)^2} - \frac{2\dddot{U}(u_i)}{3\dot{U}(u_i)}\right], \qquad \frac{dr_i}{du_i} = -\frac{r_i(u_i) - a_i(u_i)}{2r_i(u_i)}, \qquad \frac{du_i}{du_{i-1}} = \frac{r_i - a_{i-1}}{r_i - a_i}$$

- We can show that

ansatz $\frac{da_i}{du_i} = -\frac{C}{a_i^2}, \qquad r_i = a_i - 2a_i\frac{da_i}{du_i} = a_i + \frac{2C}{a_i} \qquad \Longrightarrow \qquad \frac{da_i}{du_i} = -\frac{N l_p^2}{96\pi}\frac{1}{a_i^2}$

$\Rightarrow$ Each shell emits Hawking radiation of $T_i = \frac{\hbar}{4\pi a_i}$.

Note: The existence of horizon is not essential for Hawking radiation.
Generally, particle creation can occur in a time-dependent potential.

Construction of the interior metric

Model (3/3)

- Taking continuum limit, we can obtain the interior metric:

$$ds^2 = -e^{-\frac{24\pi}{N{l_p}^2}[R(a(u))^2 - r^2]}\left[\frac{N{l_p}^2}{48\pi r^2}e^{-\frac{24\pi}{N{l_p}^2}[R(a(u))^2 - r^2]}du + 2dr\right]du + r^2 d\Omega^2$$

- If $N \gg 1$ (e.g. $O(100)$), no singularity exists: $R, \sqrt{R_{\alpha\beta}R^{\alpha\beta}}, \sqrt{R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}} \sim \frac{1}{Nl_p^2}$

  ⇒This is a self-consistent sol. of $G_{\mu\nu} = 8\pi G\langle T_{\mu\nu}\rangle$ without higher derivative corrections.

  ⇒This does not have a classical limit ($\hbar \to 0$).

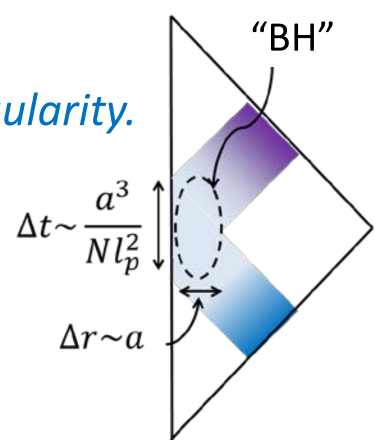
- We can check

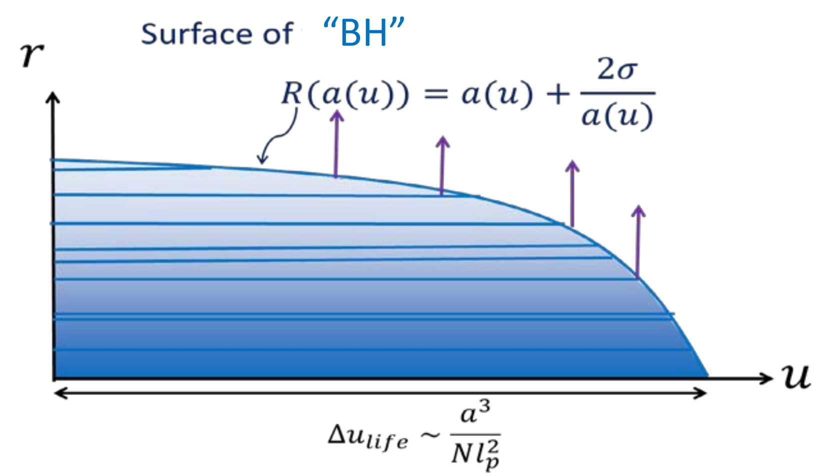
$$-\langle T^t_t\rangle = \langle T^r_r\rangle = \frac{1}{8\pi G r^2} \ll \langle T^\theta_\theta\rangle = \frac{1}{8\pi G}\frac{48\pi}{Nl_p^2}.$$

  ⇒ Energy density is positive everywhere, but dominant energy condition breaks down.

  ⇒The interior is drastically anisotropic.

⇒There is no horizon or singularity.





The metric determined by 4d Weyl anomaly

- Without using s-wave and eikonal approximation or considering many shells, we can determine the interior metric as

$$ds^2 = -\frac{2\gamma}{3(1+f)^2 r^2} e^{-\frac{3(1+f)}{2\gamma}[R(a)^2 - r^2]} dt^2 + \frac{3(1+f)^2 r^2}{2\gamma} dr^2 + r^2 d\Omega^2,$$

(assume conformal matter)

by using

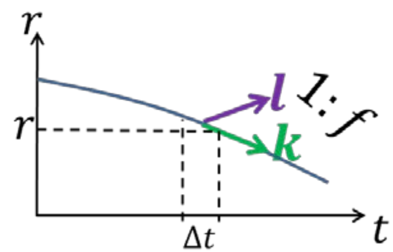
$$\begin{cases} G^\mu_\mu = 8\pi G \langle T^\mu_\mu \rangle = \gamma \mathcal{F} - \alpha \mathcal{G} \\ \langle T^{kk} \rangle : \langle T^{kl} \rangle = 1 : f \end{cases}$$

←4d Weyl anomaly

$f = O(1)$ =const.

$\mathcal{F} \equiv C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta}, \ \ \mathcal{G} \equiv R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$

$\gamma \equiv 8\pi G \hbar c, \alpha \equiv 8\pi G \hbar a.$



⇒ The 4d Weyl anomaly creates the radiation and pressure:

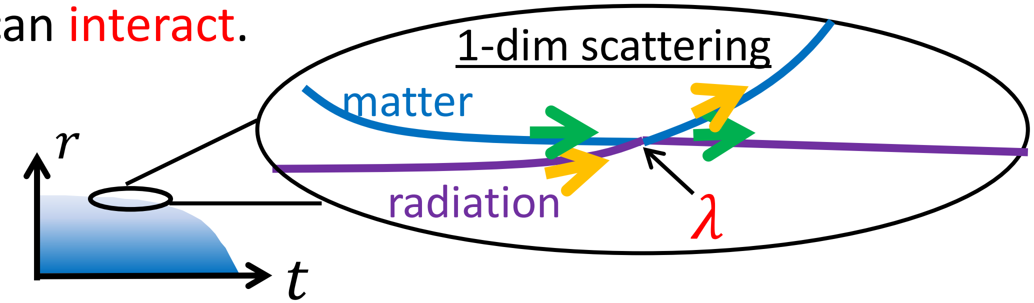
$$\sigma = \frac{8\pi l_p^2 c}{3(1+f)^2}, \qquad \langle T^\theta_\theta \rangle = \frac{1}{2} \langle T^\mu_\mu \rangle = \frac{1}{8\pi G} \frac{3}{16\pi c l_p^2}$$

⇒The interior picture is robust.

Information recovery and BH entropy

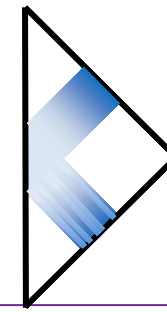
- The radiation is created inside the collapsing matter.

⇒The matter and radiation can interact.



⇒We can evaluate the scattering time scale:

$$\Delta t_{scat} \sim a \log \frac{a}{\lambda N l_p} \sim \text{scrambling time}$$

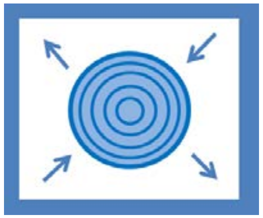


⇒The matter itself can come back. (⇒Information recovery?)

- Consider a BH that is adiabatically formed in the heat bath.

⇒We can reproduce the area law by integrating the entropy density s over volume:

$$S = \int d^3x \sqrt{h}\, s = \frac{A}{4 l_p^2}$$



Summary

The essential assumptions are

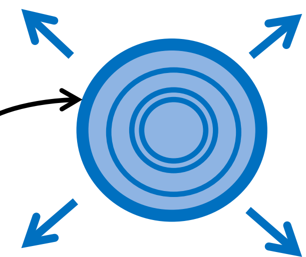
(1) Spherical symmetry

e.g. $N = O(100)$

(2) The semi-classical approximation $G_{\mu\nu} = 8\pi G\langle T_{\mu\nu}\rangle$ with $N \gg 1$

⇒The "BH" has a clear boundary at

$$R(a(t)) = a(t) + \frac{2\sigma\big(a(t)\big)}{a(t)}.$$



It looks the same as the conventional evaporating BH from the outside, but the interior is filled up with matters.

⇒The interior metric is given by

$$ds^2 = -\frac{2\sigma(r)}{r^2} e^{-\int_r^{R(a)} dr' \frac{r'}{(1+f(r'))\sigma(r')}} dt^2 + \frac{r^2}{2\sigma(r)} dr^2 + r^2 d\Omega^2.$$

Two phenomenological functions

$$\frac{da}{dt} = -\frac{2\sigma(a)}{a^2}, \qquad \langle T^{kk}\rangle : \langle T^{kl}\rangle = 1 : f(\mathrm{r})$$

⇒There is no horizon or singularity.

This is the "BH".

Thank you very much!