

Influence of a plasma on light propagation in general relativity

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1. Light propagation in a non-magnetised, pressure-free plasma
2. Travel time and deflection of light in a plasma
Application: Solar corona, plasma around black hole
3. Geometry of light bundles in a plasma, generalised Sachs equations
Application: Robertson-Walker cosmology with plasma

Light rays on a general-relativistic spacetime with metric $g_{\mu\nu}(x)$:

$$\dot{x}^\mu = \frac{\partial H(x, p)}{\partial p_\mu}, \quad \dot{p}_\mu = -\frac{\partial H(x, p)}{\partial x^\mu}, \quad H(x, p) = 0$$

In vacuo:

$$H(x, p) = \frac{1}{2} g^{\mu\nu}(x) p_\mu p_\nu$$

Light rays are lightlike geodesics of the spacetime metric $g_{\mu\nu}$

In a non-magnetised pressure-free plasma:

$$H(x, p) = \frac{1}{2} \left(g^{\mu\nu}(x) p_\mu p_\nu + \omega_p(x)^2 \right),$$

plasma frequency: $\omega_p(x)^2 = \frac{e^2}{\epsilon_0 m_e} N(x)$

e : charge of the electron, m_e : mass of the electron

$N(x)$: number density of the electrons

Light rays are timelike geodesics of the conformally rescaled

metric $\tilde{g}_{\mu\nu} = \omega_p^{-2} g_{\mu\nu}$

Rigorous derivation from Maxwell's equation, even for magnetised pressure-free plasma:

R. Breuer, J. Ehlers: Proc. Roy. Soc. London, A 370, 389 (1980), A 374, 65 (1981)

for non-magnetised pressure-free plasma:

VP: "Ray Optics, Fermat's Principle and Applications to General Relativity" Springer (2000)

A plasma is a dispersive medium; propagation of light rays depend on the frequency $\omega = -p_\mu U^\mu$

For a pressure-free non-magnetised plasma, only the plasma frequency matters, not the 4-velocity of the electrons

Light rays are characterised by a Lorentz invariant index of refraction

$$n(x, \omega)^2 = 1 - \frac{\omega_p(x)^2}{\omega^2}.$$

J. Synge: "Relativity: The General Theory", North-Holland (1960)

Influence of a plasma is measurable:

- **on the travel time of radio signals**
 - in the interstellar medium (dispersion measure of pulsar signals)
 - in the Solar corona (correction to Shapiro time delay)
- **on the deflection of radio signals**
 - in the Solar corona (used for measuring the plasma density function)

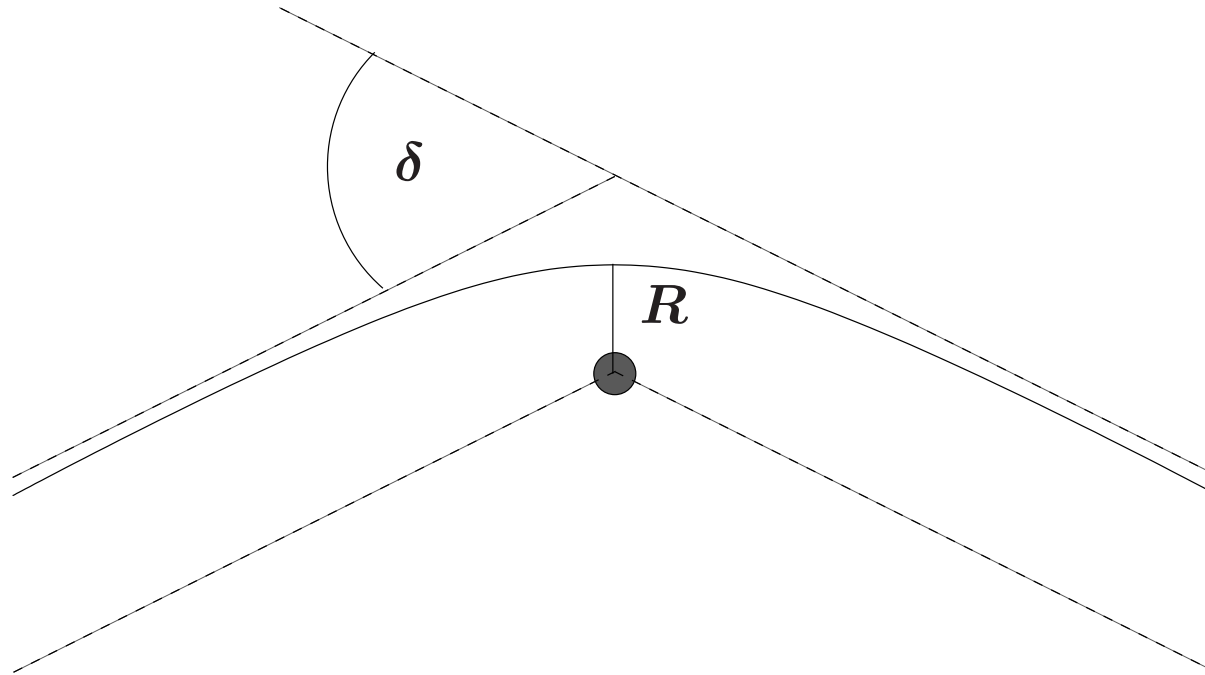
Theory (weak field):

D. O. Muhleman and I. D. Johnston: Phys. Rev. Lett. 17, 455 (1966)

Exact formula for light bending in a plasma on Schwarzschild spacetime:

$$\pi + \delta = 2 \int_R^\infty \left(\frac{r^2 \left(\frac{r}{r-2m} - \frac{\omega_p(r)^2}{\omega_0^2} \right)}{R^2 \left(\frac{R}{R-2m} - \frac{\omega_p(R)^2}{\omega_0^2} \right)} - 1 \right)^{-1/2} \frac{dr}{\sqrt{r} \sqrt{r-2m}}$$

VP: “Ray optics, Fermat’s principle and applications to general relativity” Springer (2000)



Generalisation to arbitrary spherically symmetric and static space-time

VP, O. Yu. Tsupko, G. S. Bisnovatyi-Kogan: Phys. Rev. D 92, 104031 (2015)

Applications to black holes, wormholes, ...

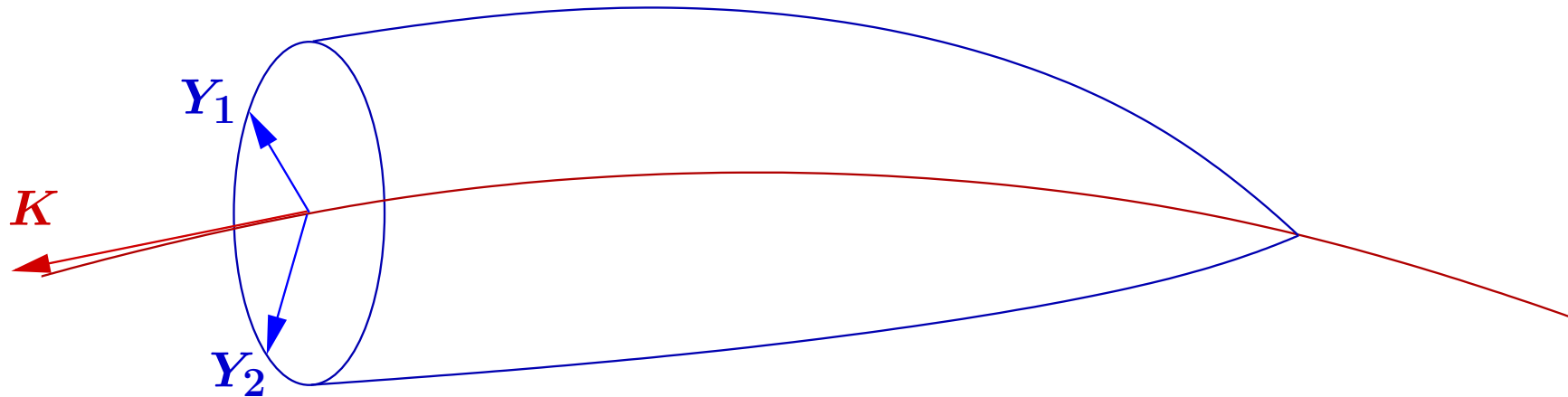
In particular: Influence of a plasma on the shadow of a Schwarzschild black hole

see talk by Oleg Tsupko, B1, Thu 2:15pm

Geometry of light bundles:

K. Schulze-Koops, D. Schwarz, VP : in preparation

Light ray in plasma, parametrised such that $g(K, K) = -\omega_p^2$



Bundle spanned by connecting vectors (Y_1, Y_2) to neighbouring light rays with $g(Y_A, K) = 0$

Sachs basis (E_1, E_2) : $g(E_A, E_B) = \delta_{AB}$, $g(E_A, K) = 0$

$$\tilde{\nabla}_K \frac{E_1}{\sqrt{\tilde{g}(E_1, E_1)}} = \tilde{\nabla}_K \frac{E_2}{\sqrt{\tilde{g}(E_2, E_2)}} = 0$$

$$Y_A = D_A{}^B E_B + Y_A^\perp, \quad \mathbb{D} = (D_A{}^B)$$

$$\mathbb{D}^{-1}\dot{\mathbb{D}} = \begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix} + \begin{pmatrix} \sigma_1 & \sigma_2 \\ \sigma_2 & -\sigma_1 \end{pmatrix} + \frac{\theta}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Optical scalars: $\varrho = \theta + i\omega$, $\sigma = \sigma_1 + i\sigma_2$

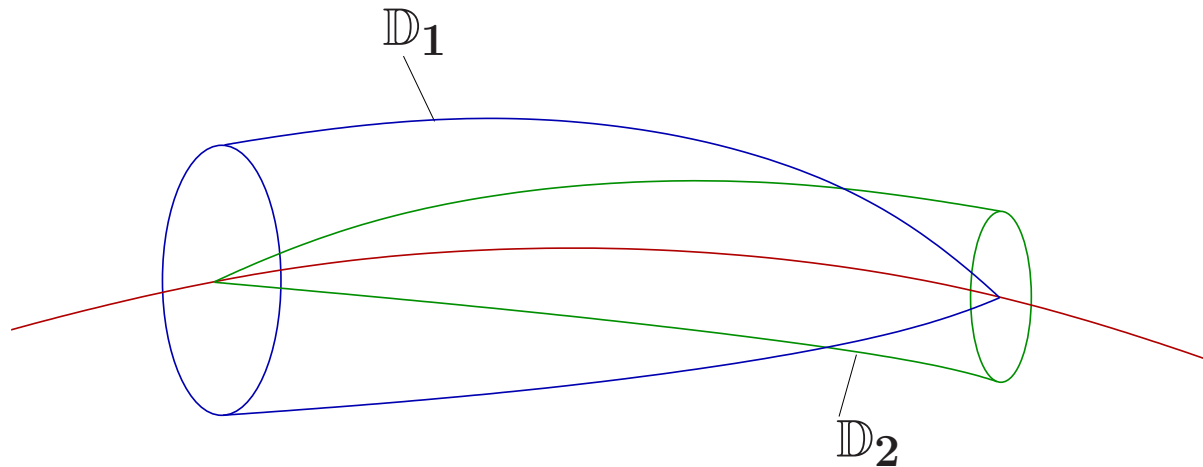
Generalised Sachs equations:

$$\begin{aligned} \dot{\varrho} + \varrho^2 + |\sigma|^2 &= \frac{1}{2}\text{Ric}(K, K) + \frac{1}{2} \left(g(E_1, C(K, E_1, K)) + g(E_2, C(K, E_2, K)) \right) \\ &\quad - \frac{1}{4} \omega_p^2 \left(\text{Ric}(E_1, E_1) + \text{Ric}(E_2, E_2) \right) + \frac{1}{3} \omega_p^2 R \\ &\quad + 2\nabla^\mu \omega_p \nabla_\mu \omega_p + \omega_p \nabla^\nu \nabla_\nu \omega_p - \frac{1}{2} \nabla_\mu \nabla_\nu \omega_p (E_1^\mu E_1^\nu + E_2^\mu E_2^\nu) \\ \dot{\sigma} + 2\theta\sigma &= \frac{1}{2} g(E_1 + iE_2, C(K, E_1 + iE_2, K)) \\ &\quad - \frac{1}{4} \omega_p^2 \text{Ric}(E_1 + iE_2, E_1 + iE_2) \\ &\quad + \left(\nabla_\mu \omega_p \nabla_\nu \omega_p - \frac{1}{2} \omega_p \nabla_\mu \nabla_\nu \omega_p \right) (E_1^\mu + iE_2^\mu) (E_1^\nu + iE_2^\nu) \end{aligned}$$

Application to cosmology:

Spatially homogeneous plasma on Robertson-Walker spacetime produces no image deformation (no shear), but it influences the distance measures

Modified reciprocity theorem gives relation between area distance and luminosity distance



$$d_L = (1 + z)(1 + z_\lambda)d_A$$

$$1 + z = \frac{\omega_E}{\omega_R}, \quad 1 + z_\lambda = \frac{\lambda_R}{\lambda_E}$$