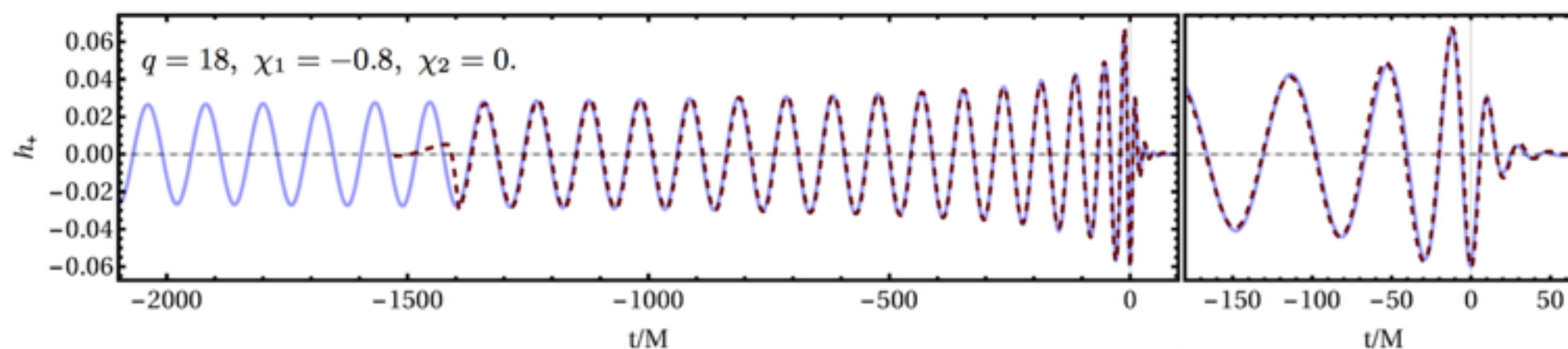


# Phenomenological waveform models for the advanced detector era



Phenom\* models have been used to analyse first detections - what are these models, and how are we improving them.

Goal: synthesize **inspiral-merger-ringdown** models of the complete WF of Compact Binary Coalescence from pN/EOB, NR, BH perturbation theory, Kerr geodesics, self-force.

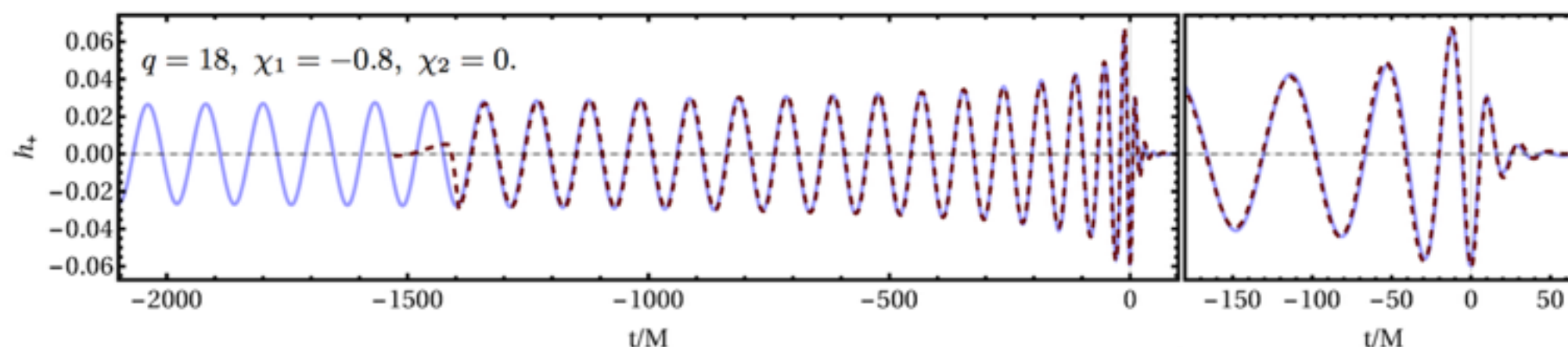


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**open source  
in LAL**

Contributors: M. Hannam, M. Pürrer, S. Khan, F. Ohme, A. Bohé, X. Jiménez, P. Schmidt, D. Keitel

PhenomD: non-precessing models

Frequency-domain gravitational waves from non-precessing black-hole binaries -  
I. New numerical waveforms and anatomy of the signal  
II. A phenomenological model for the advanced detector era

SH, S Khan, M Hannam, M Pürrer, F Ohme, X Jiménez Forteza, A Bohé, PRD  
2016

S Khan, SH, M Hannam, F Ohme, M Pürrer, X Jiménez Forteza, A Bohé, PRD  
2016



PhenomP: precessing models

M Hannam, P Schmidt, A Bohé, L Haegel, S Husa, F Ohme, G Pratten, M  
Pürrer. Phys. Rev. Lett. 113, 151101 (2014), detailed PhenomP (version 2)  
paper coming up

Systematic model building and double spin upgrades:  
New work with X Jiménez Forteza & D Keitel

# Phenomenological modelling of IMR waveforms

---

- Key design ideas [\[alternative: Effective One Body -> A Bohé's talk\]](#)
  - “phenomenological”: look at waveforms and describe what we see.
  - Frequency domain: matched filter calculations in Freq. domain.
  - Explicit expression in terms of elementary functions -> fast, simple.  
[\[fast without ROM acceleration\]](#)
- Start with  $l=|m|=2$  spherical harmonic mode (in co-rotating frame)
- Model directions in parameter space in order of importance.
  - Non-spinning (PhenomA)
  - Single effective spin, neglect spin difference effects (PhenomB/C/D).
  - Leading precession effects via PN, no NR calibration (PhenomP)
  - In progress: full double spin model
    - systematic model building procedure 
    - final state and peak amplitude -> Xisco's talk Monday 

# A simple model for precession: PhenomP

---

- **Orbital time scale  $\ll$  precession time scale**

=> Co-rotating frame: radiated  $\vec{L}$ , energy & phasing essentially unaffected by precession

=> dominated by  $\vec{S}_i \cdot \vec{L}$

- Spherical harmonic mode structure in standard frame corresponds to non-precessing case

-> “twisting up” aligned spin model with “post-Newtonian” Euler angles works well

Fourier domain PhenomP construction:

[Schmidt+ PRD 2012, Hannam+ PRL 2014]


Time domain EOB version: Pan+ PRD 89, 2014]

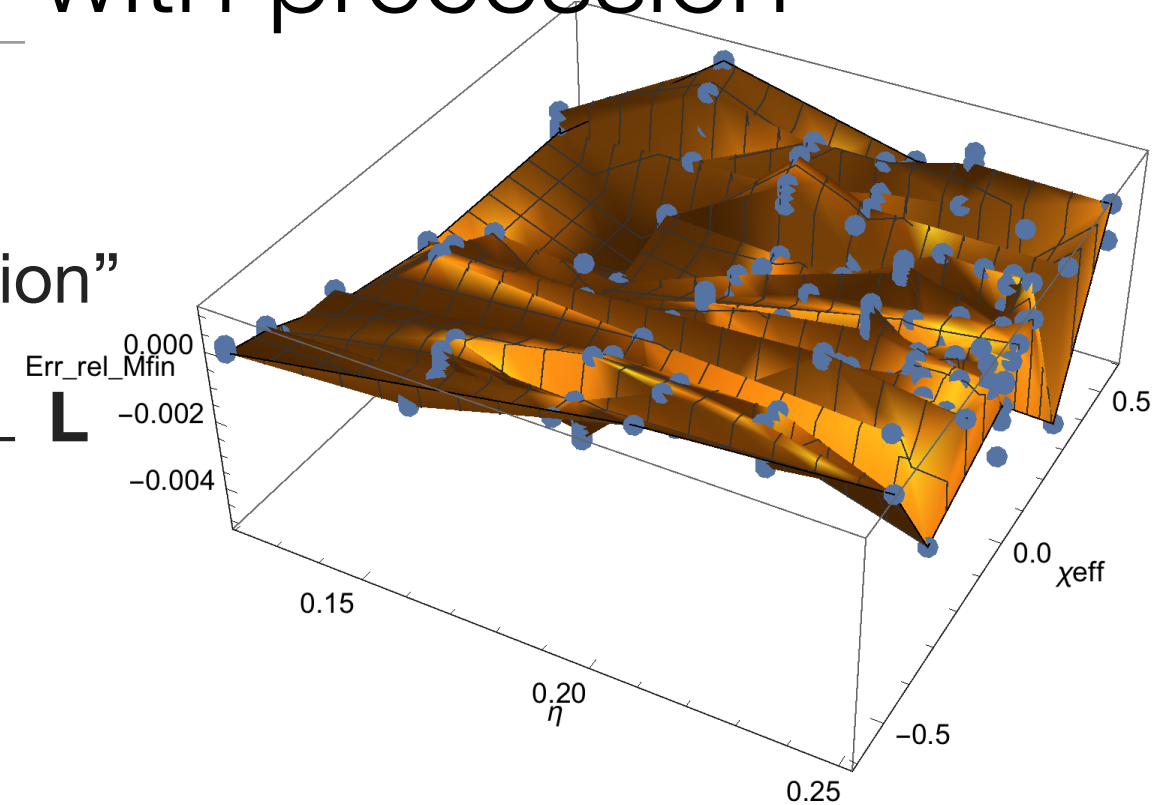
## PhenomP shortcomings:

- Not tuned to actual precessing NR waveforms.
- Incomplete:
  - Does not handle aligned/anti-aligned instability [Gerosa+ PRL (2015)]
  - Does not include symmetry-breaking “superkick recoil” effect



# Final spin and radiated energy with precession

- Based on PhenomP approximation:
  - emission in comoving frame ~ “no precession”
  - ~ preserve total spin projections unto  $\parallel$  &  $\perp$   $\mathbf{L}$
  - => radiated energy should depend only weakly on precession. 



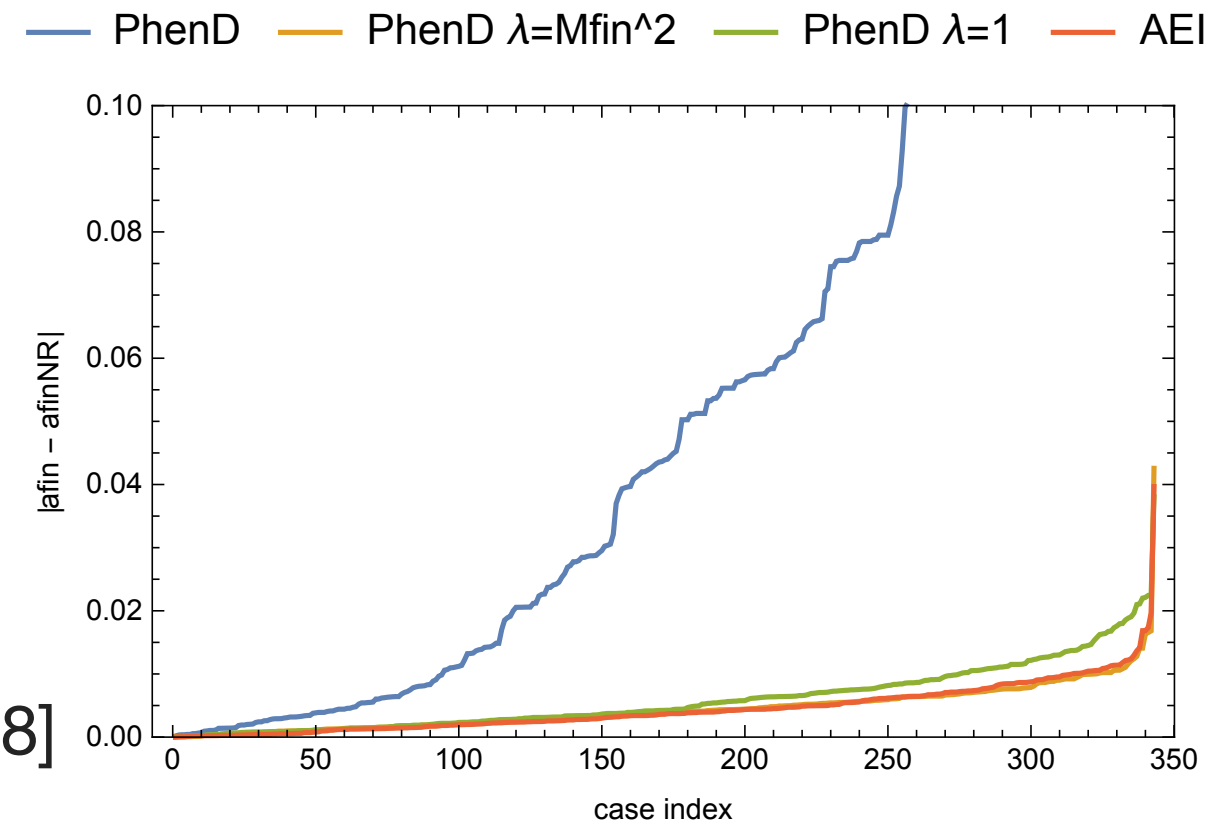
- Final spin:

$$|a_{fin}| = \sqrt{S_{\perp}^2 \frac{\lambda^2}{M_{fin}^2} + a_{fin}^{\parallel 2}}$$

- choose “fudge parameter”

$$\lambda = M_{fin}^2$$

- as in 2007 AEI fit [Rezzolla+,PRD78,2008]

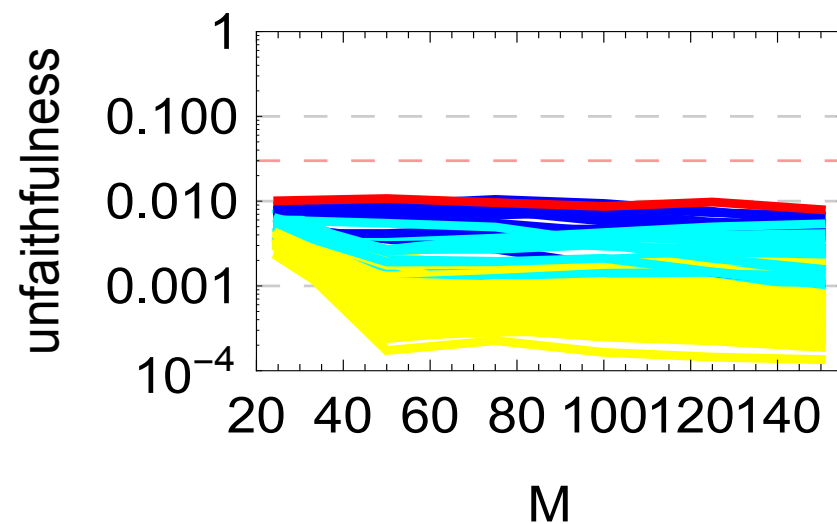


# PhenomPv2 used in O1 BBH parameter estimation

- relies on accurate non-precessing model!
- 2010 PhenomC replaced by 2015 PhenomD model
- twisted up with 2PN PN+SPA Euler angles (detailed paper in prep.)

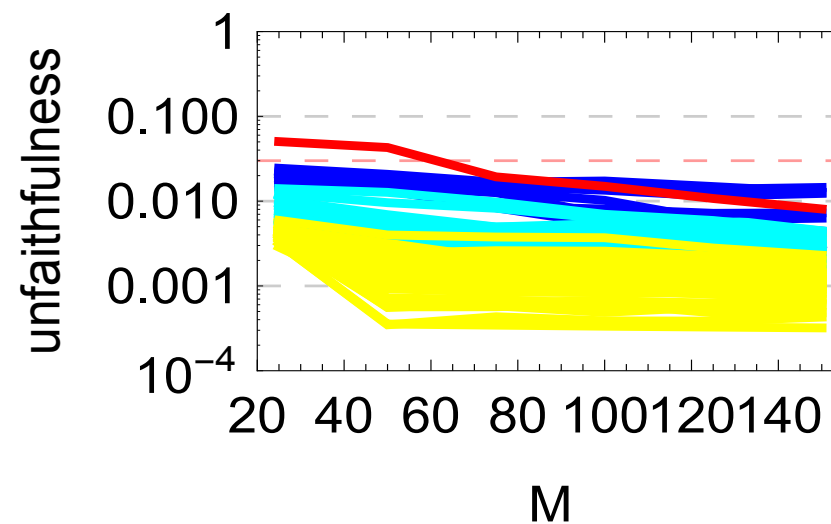
Full catalog (72 waveforms)

$$\theta = \pi/12$$



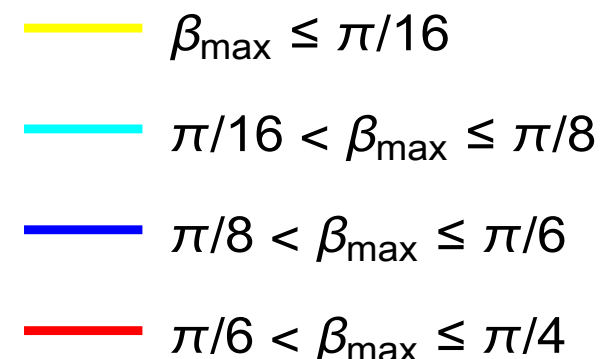
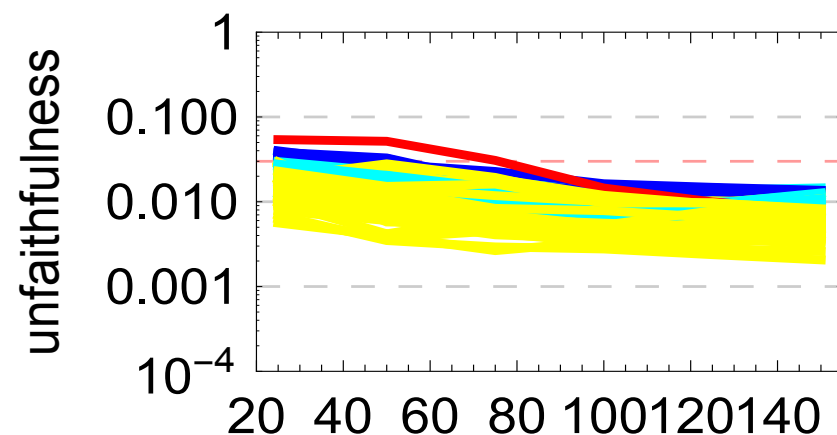
Full catalog (72 waveforms)

$$\theta = \pi/3$$



Full catalog (72 waveforms)

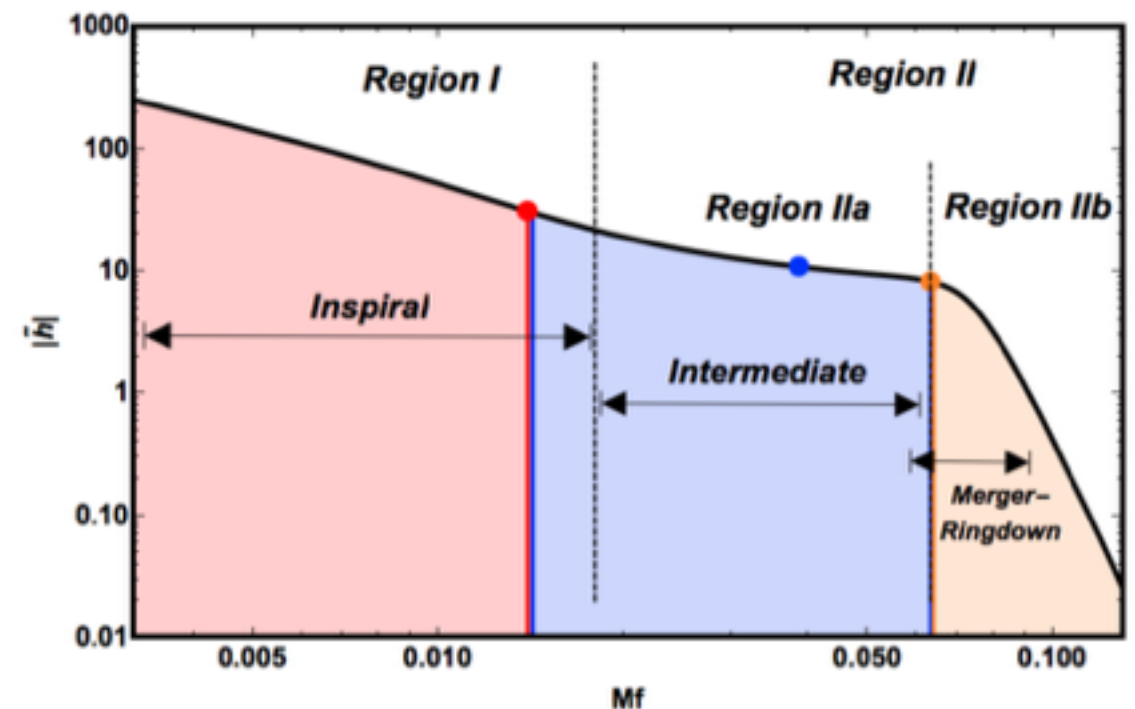
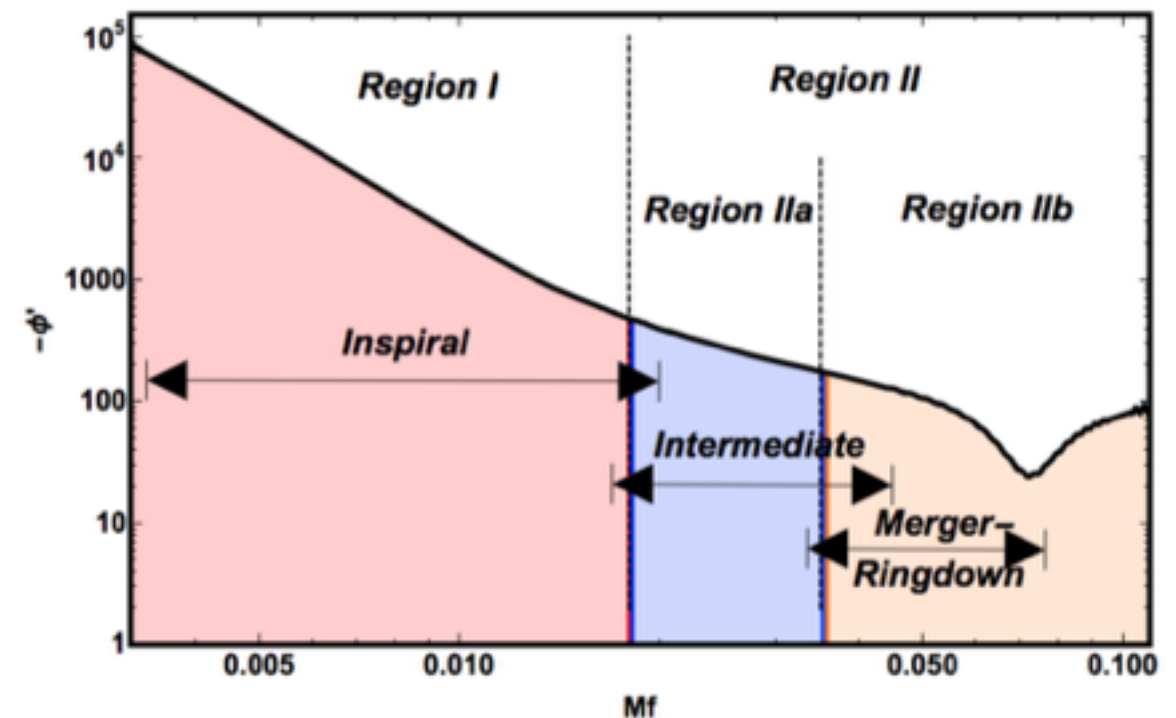
$$\theta = \pi/2$$



# Splitting into amplitude/phase & frequency regions

## Divide and conquer:

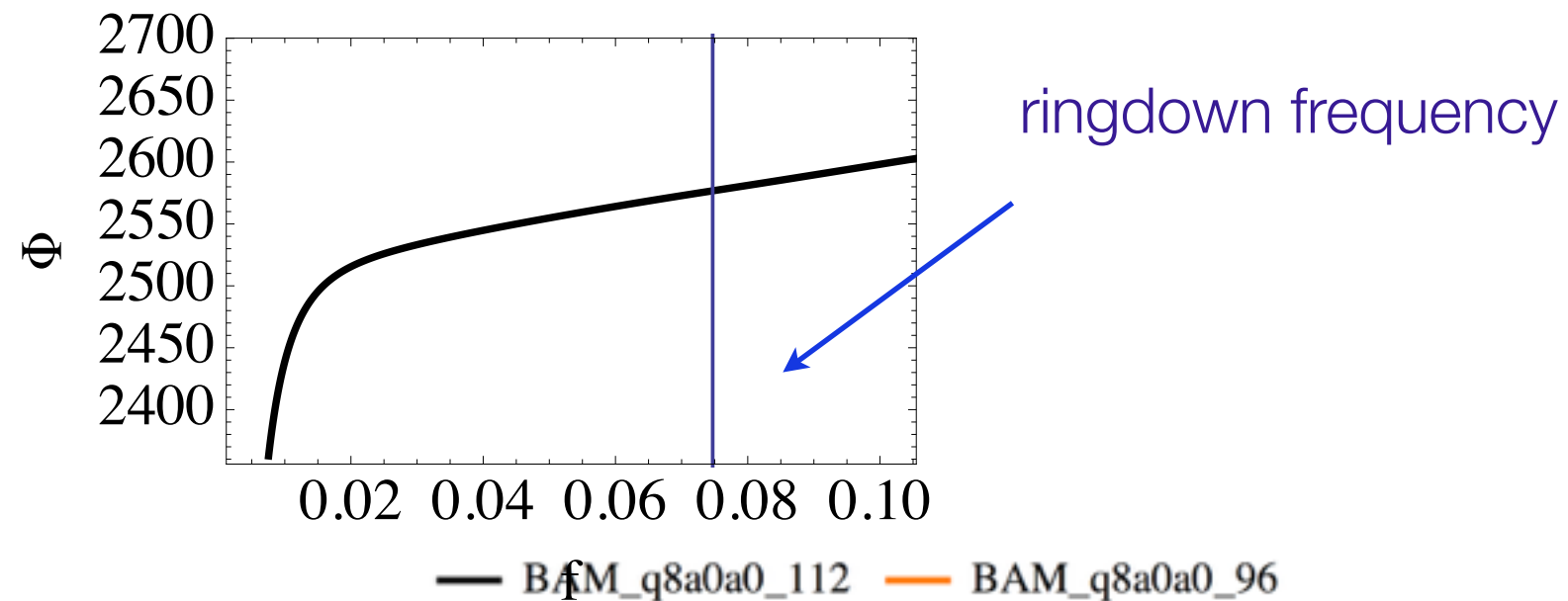
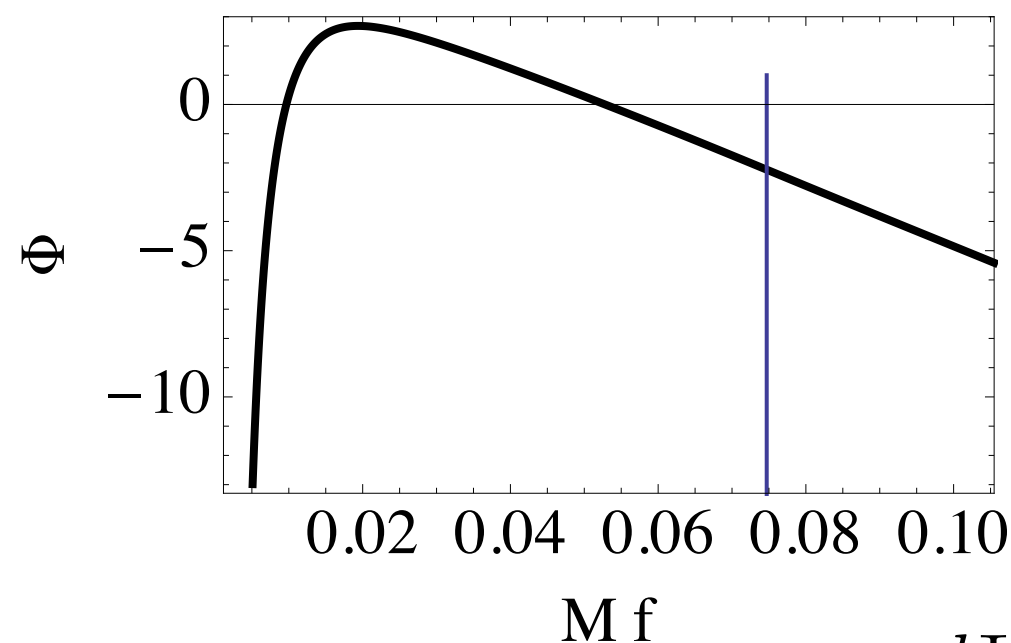
- Split waveform into amplitude and phase, model simple non-oscillatory functions.
- Simplicity of modelling increases with the number of frequency-regions.
- Simplest: tens of points, cubic spline.
- Our choice - 3 regions:
  - inspiral (use PN intuition)
  - merger-ringdown (use QNM intuition)
  - intermediate





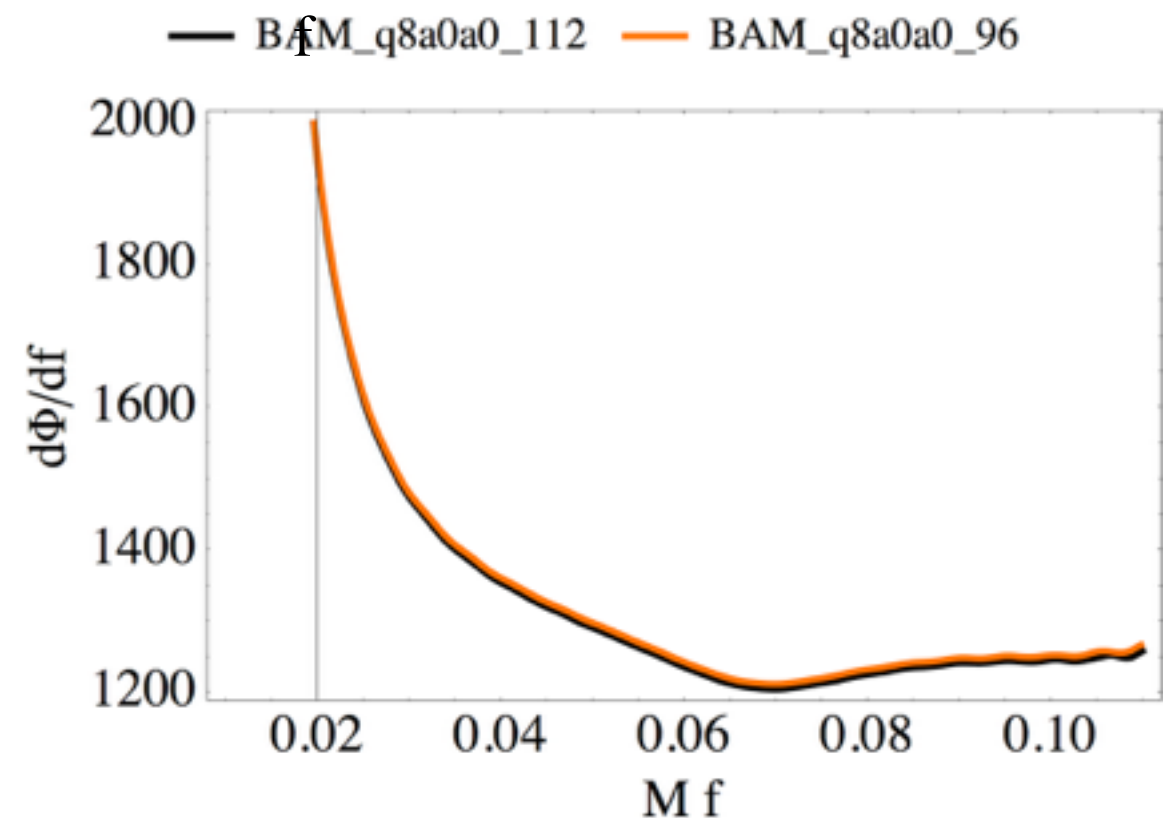
# Example: Modelling the Fourier domain phase

- Freedom in initial phase & time shift:  $\Phi(f) \rightarrow \Phi(f) + \Phi_0 + 2\pi t$



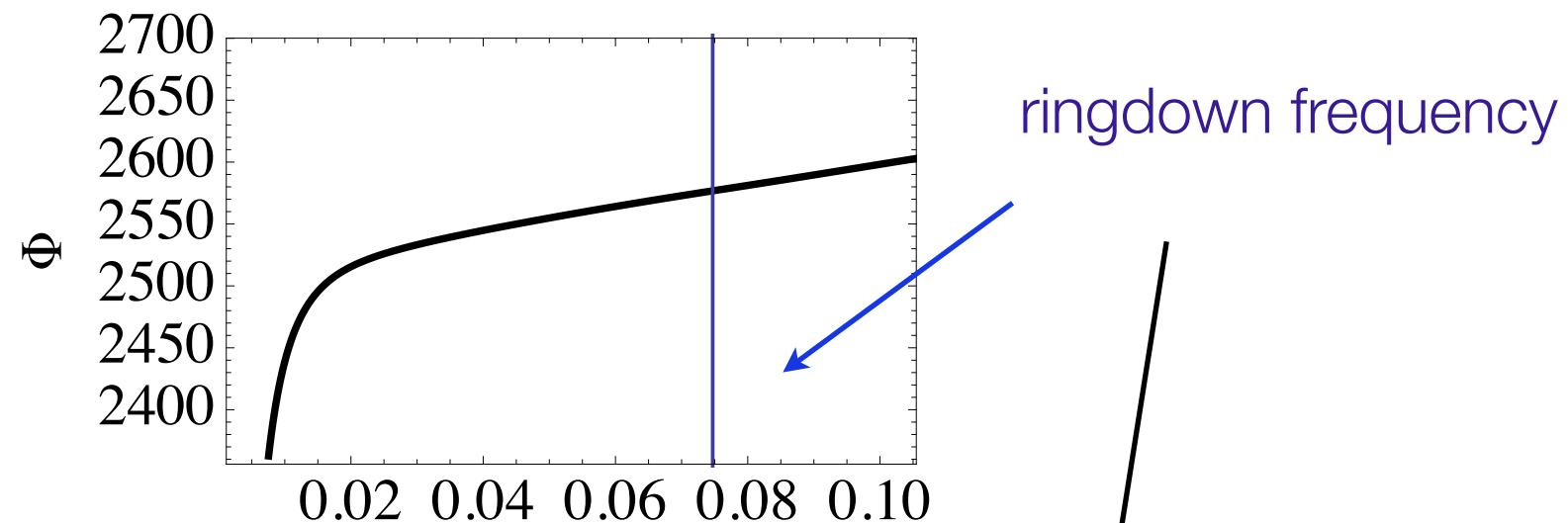
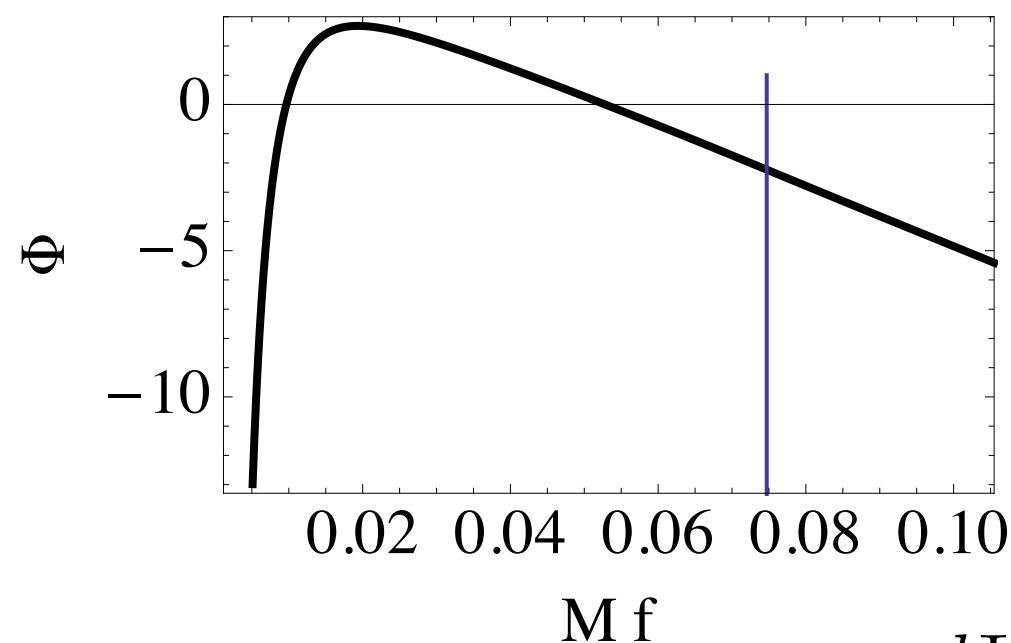
- Look at first derivative:  $\frac{d\Phi(f)}{df}$ 
  - 2<sup>nd</sup> derivative often too noisy.
- Can you spot the ringdown frequency?
- MRD-Ansatz:

$$\phi'_{MR} = \alpha_1 + \sum_{i=2}^n \alpha_n f^{-p_n} + \frac{a}{f_{damp}^2 + (f - f_{RD})^2}$$



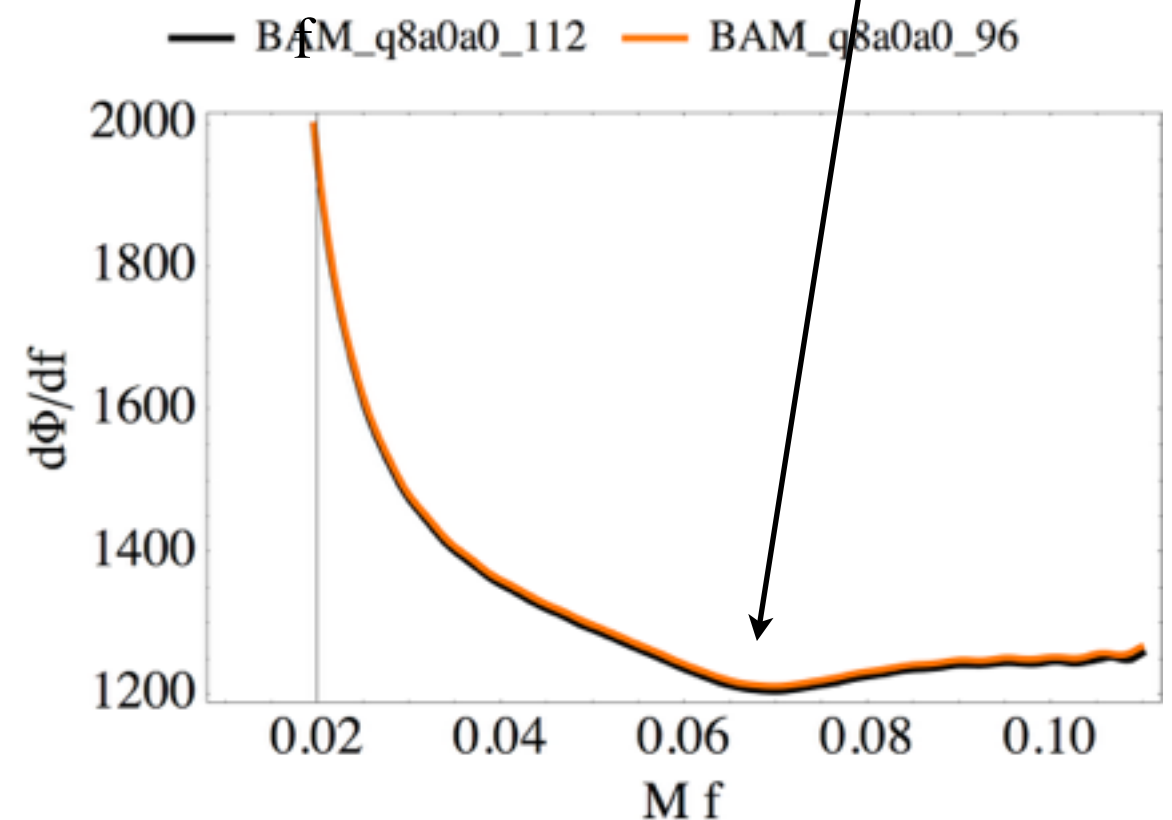
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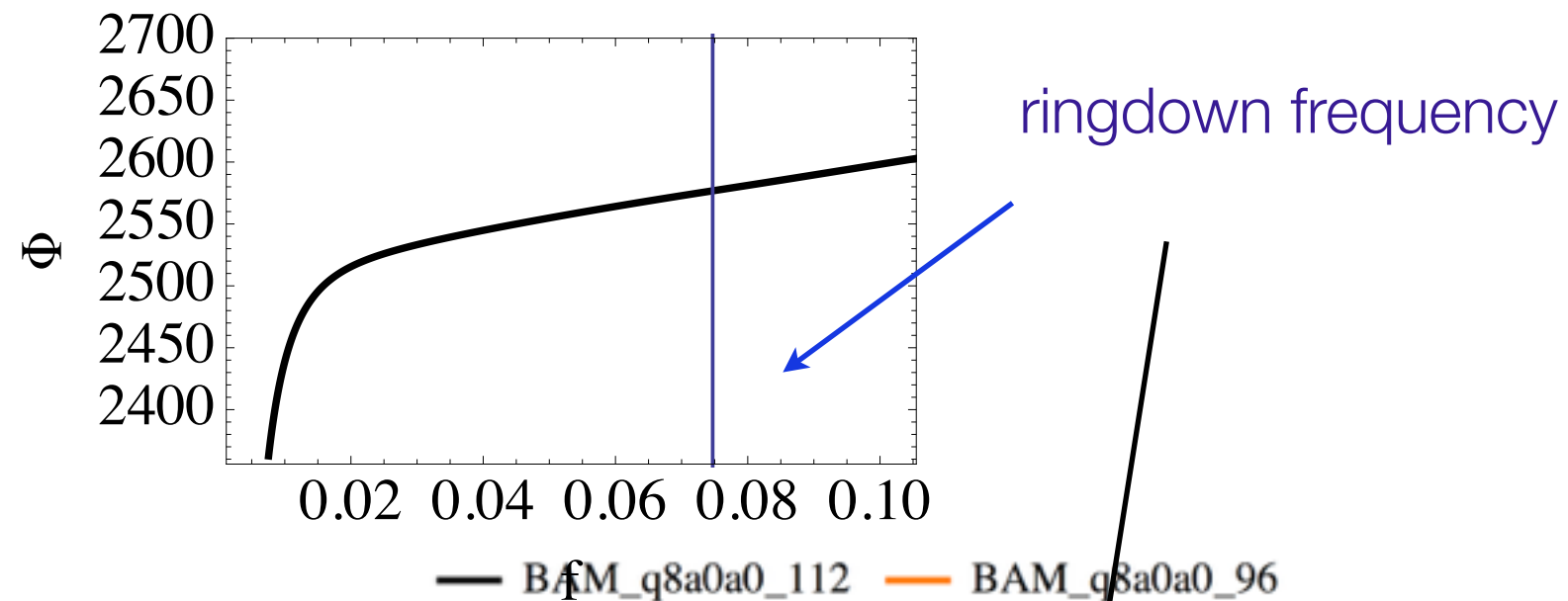
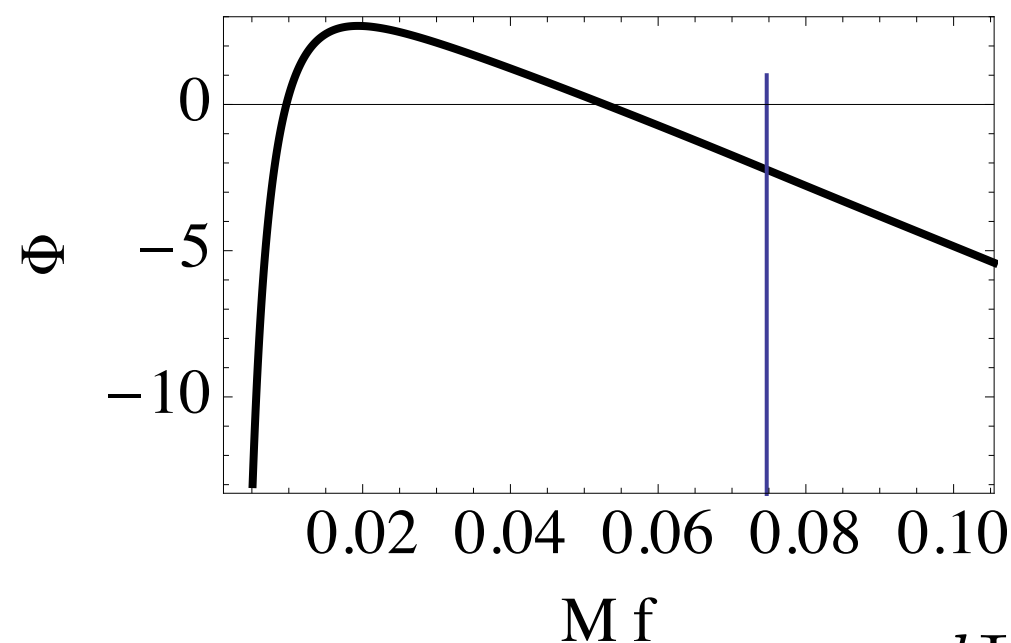
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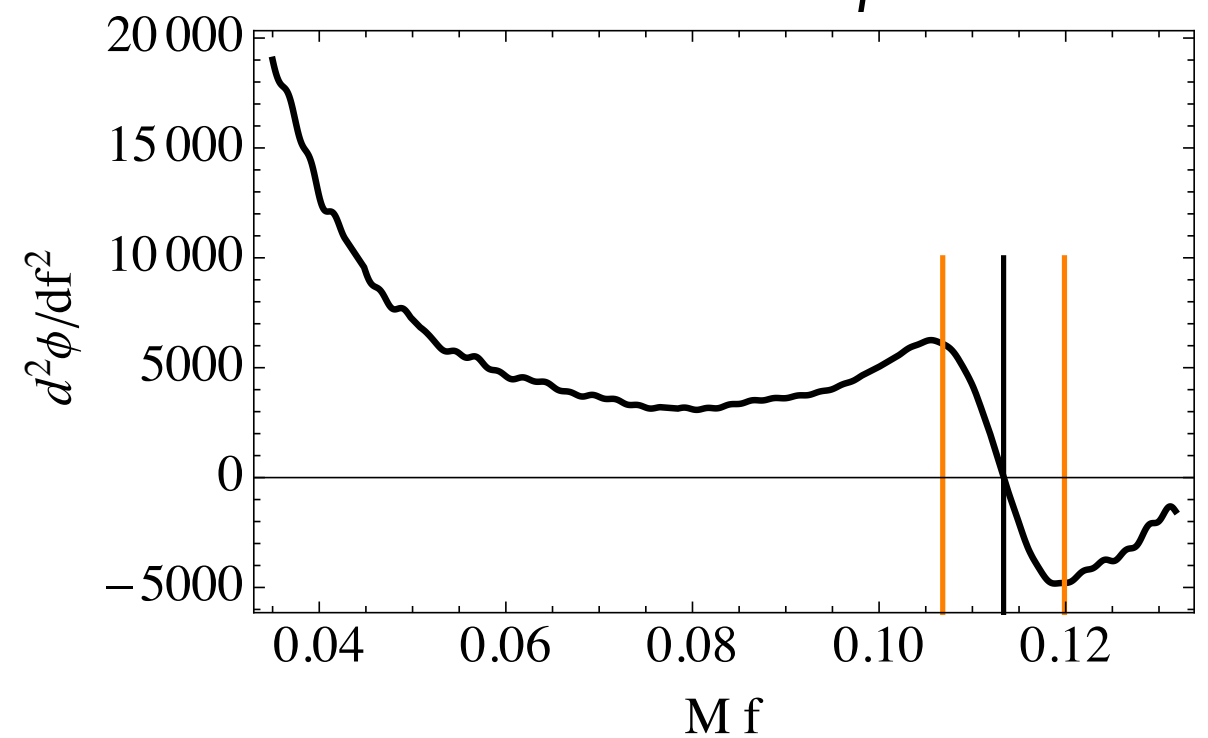
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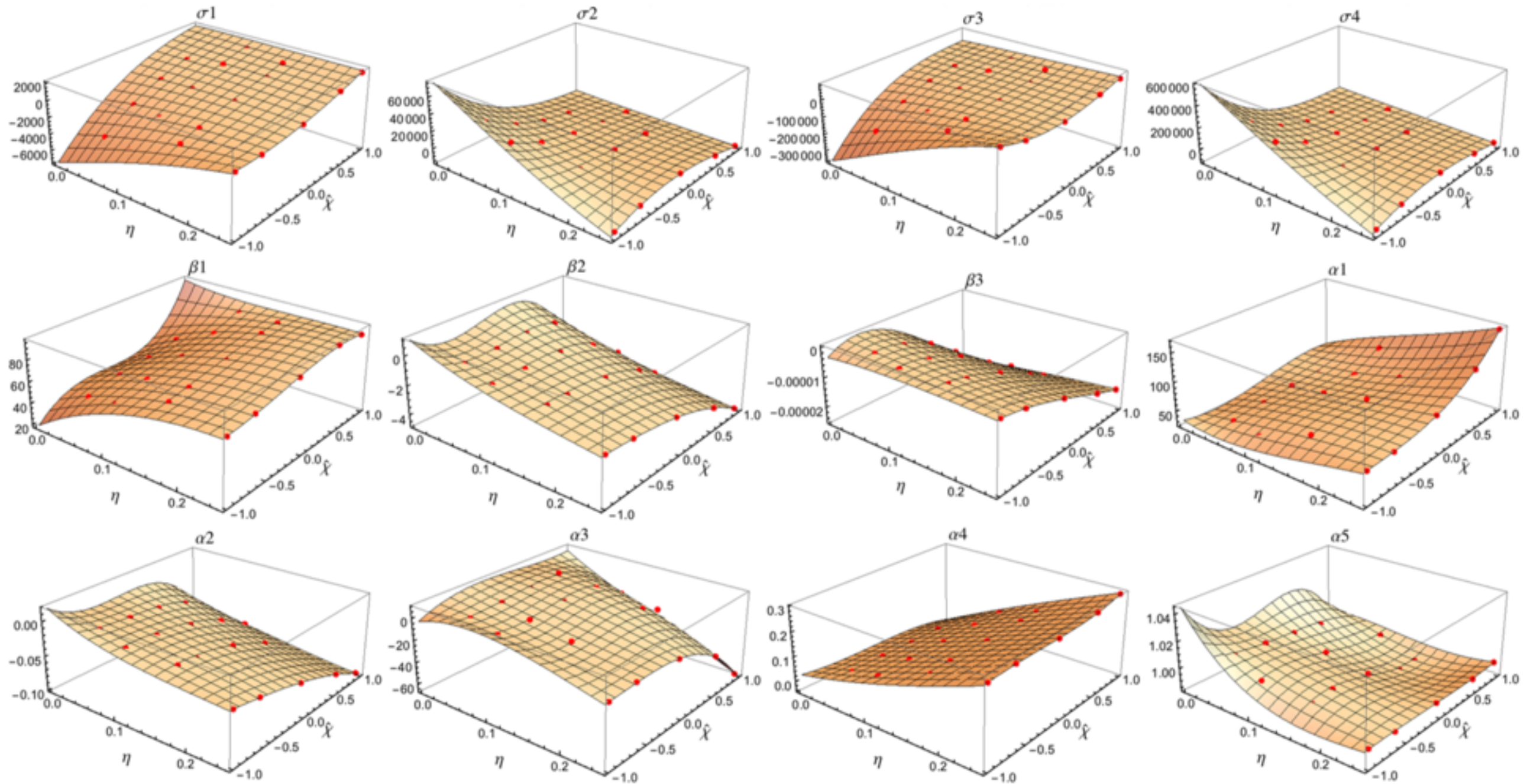


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# Phase coefficients as functions of $\eta, \hat{\chi}$



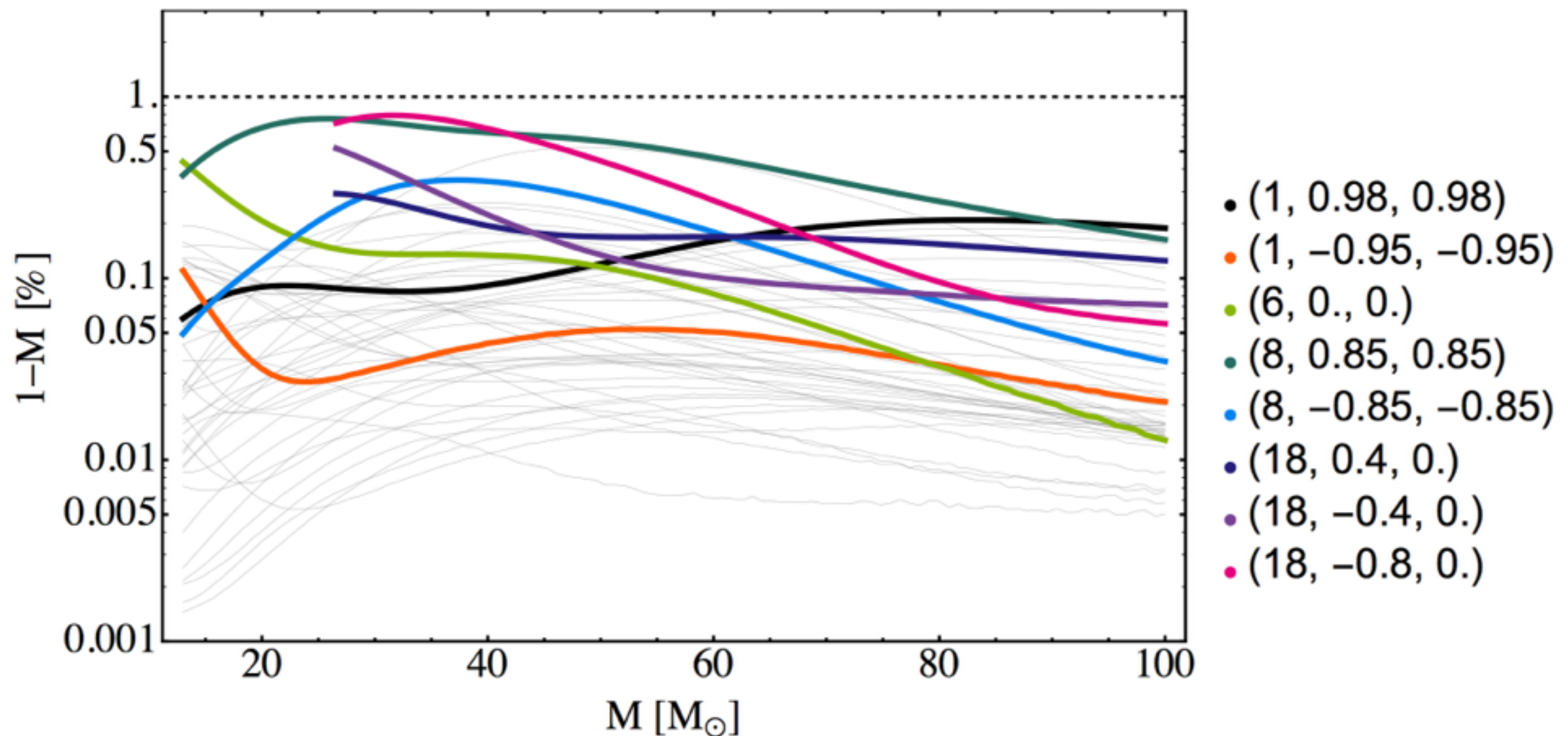
New:

- more WFs: 19  $\rightarrow$  80+, include  $m_1/m_2=18$ , spins 0.8,0
- use all available WFs with weights depending on error bars.



# PhenomD mismatches against all (48) hybrids

early aLIGO noise curve, low freq. cutoff @ 30 Hz

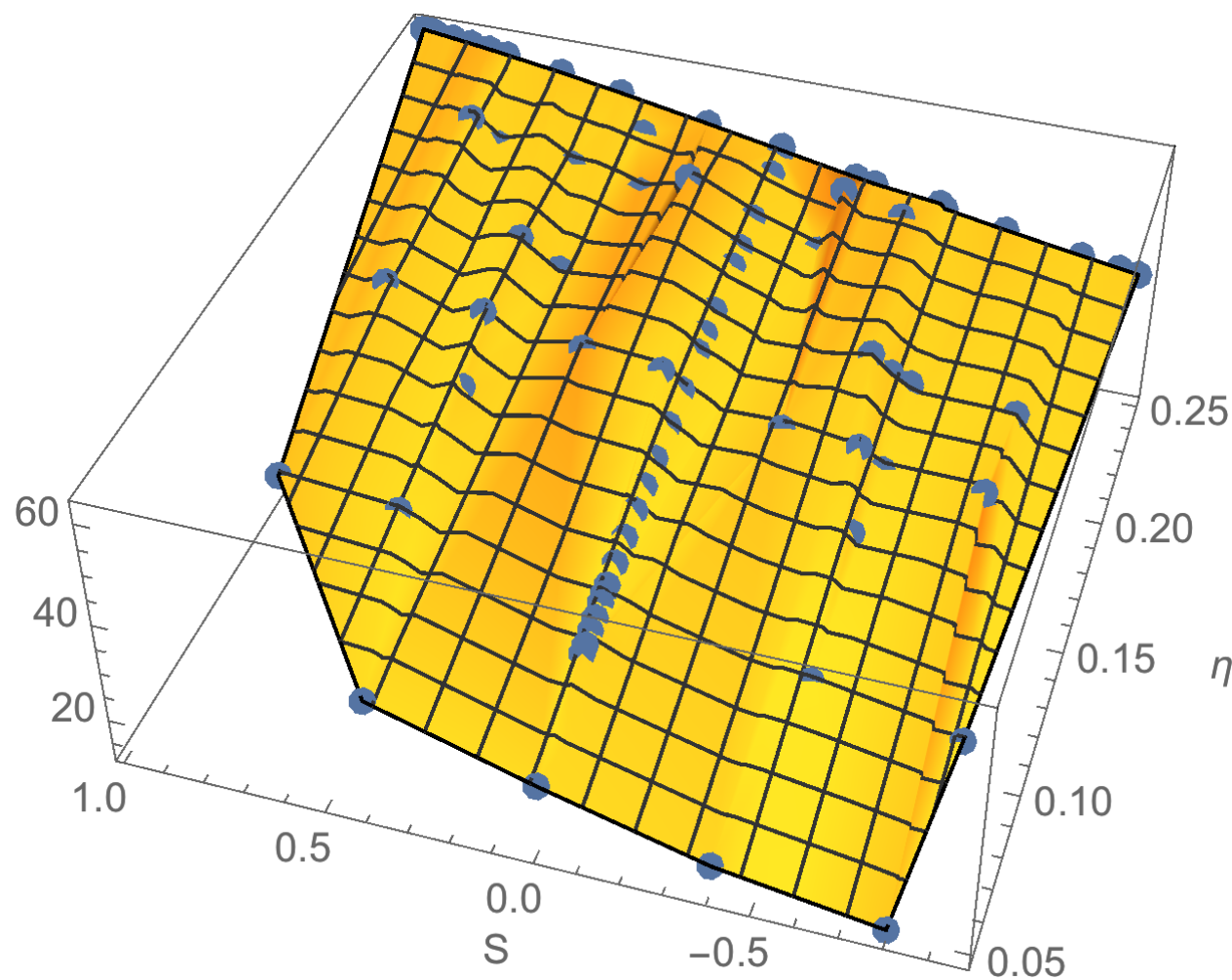




# Systematic fitting example: control point difference in merger regime

---

Better conditioned than basis coefficients: fit values of value at fixed frequencies, and differences between values.



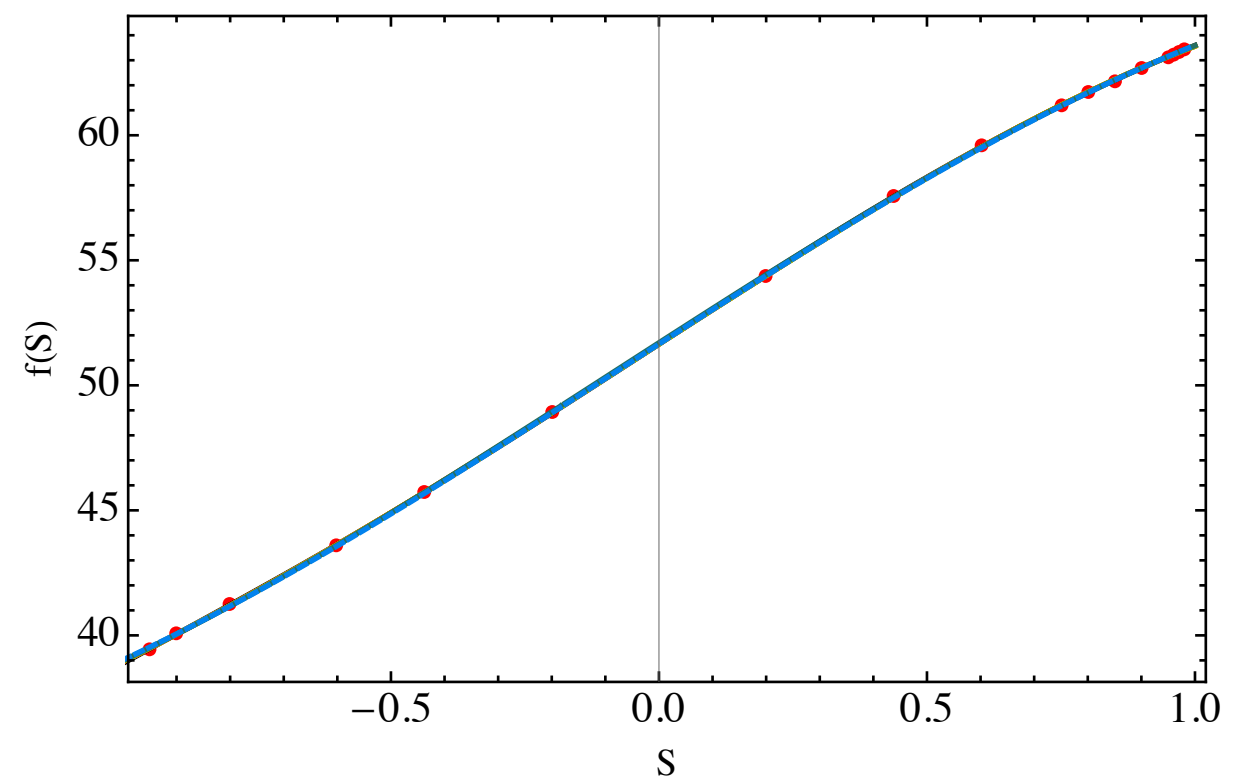
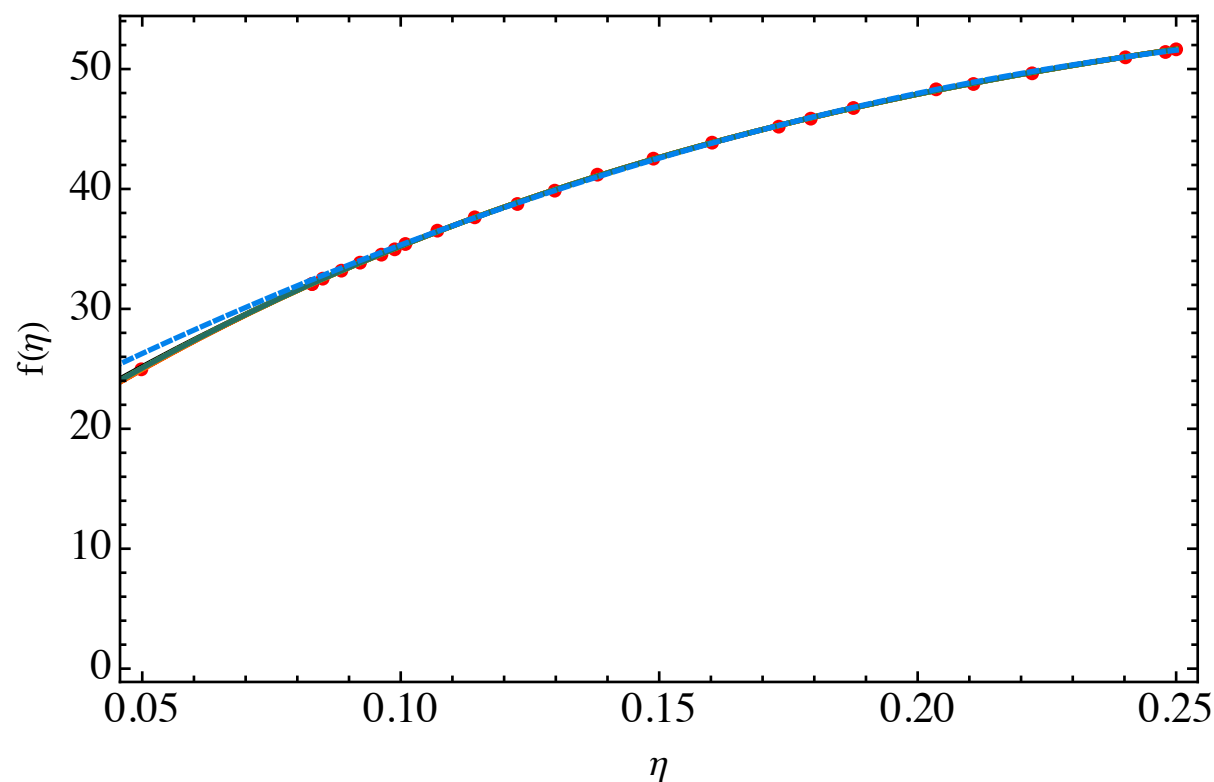
- eff. spin parameter  $S$  works reasonably well
- spin diff. can not be neglected for good fit.
- accurate polynomial fits may require high order.
- a good ansatz may only require very few coefficients in some direction.
- To avoid confusion with further subdom. effects (higher modes, precession), want accurate representation of spin difference effect.

# Systematic fitting example: control point difference in merger regime

---

Start with selecting best fits to 1D problems with dense data:

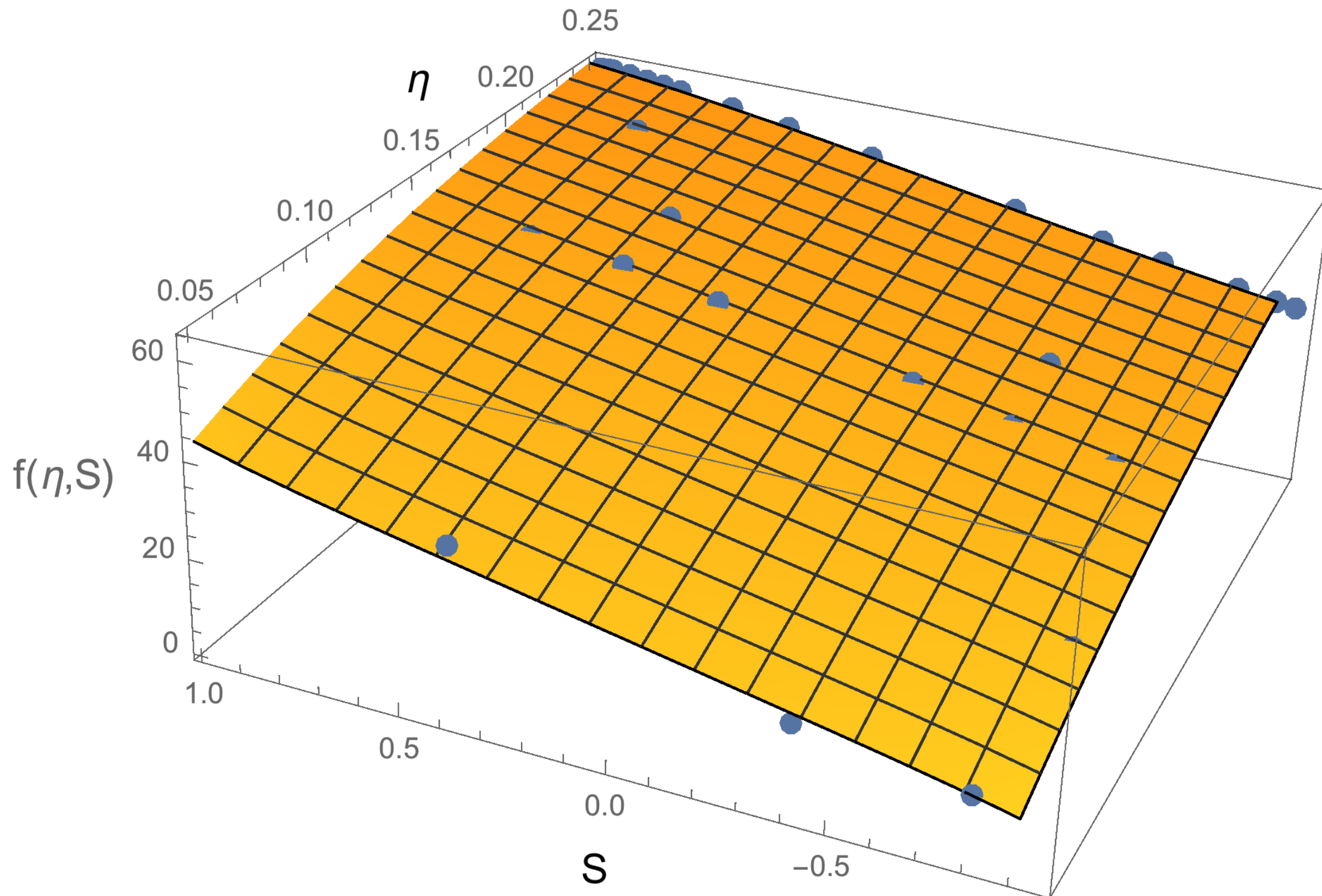
- non-spinning
- equal mass equal spin



Use BIC, AIC, AICc together with residuals to find best fit.

# Equal-spin 2D fit.

---



# Unequal spin fit

- Fit unequal spins term of the form:

$$X = X_{Eq} + f(\eta) (\chi_1 - \chi_2) + \dots$$

- Guess ansatz for  $f(\eta)$  from inspecting data:

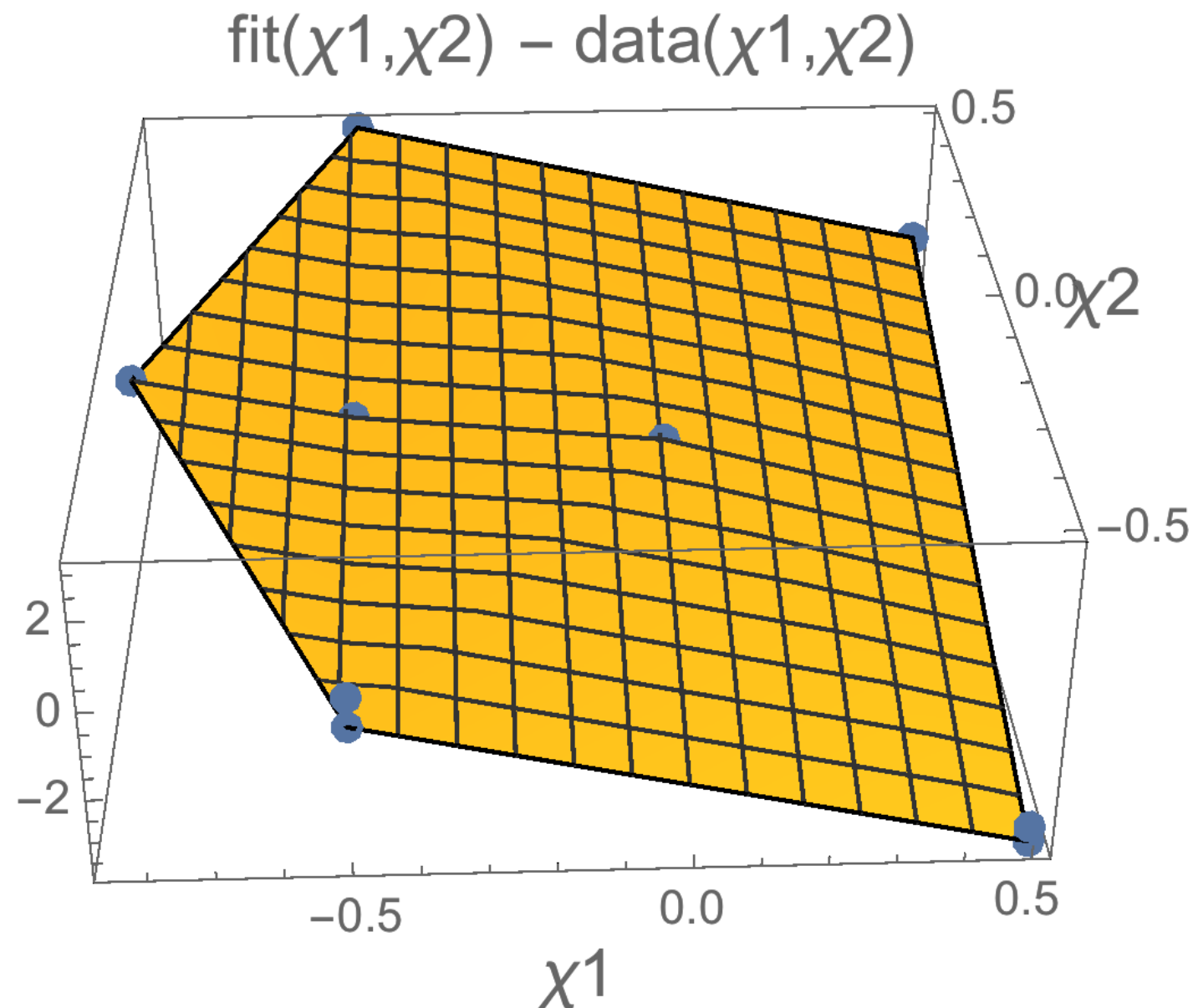
$$f(\eta) = a_0 \eta^p (1 - 4\eta)^q$$

- Consistency check:

- 1D fit vs.  
direct fit to all data

- to be completed:

tune unequal spin fits  
for  $\sim 30$  PhenomD coefficients



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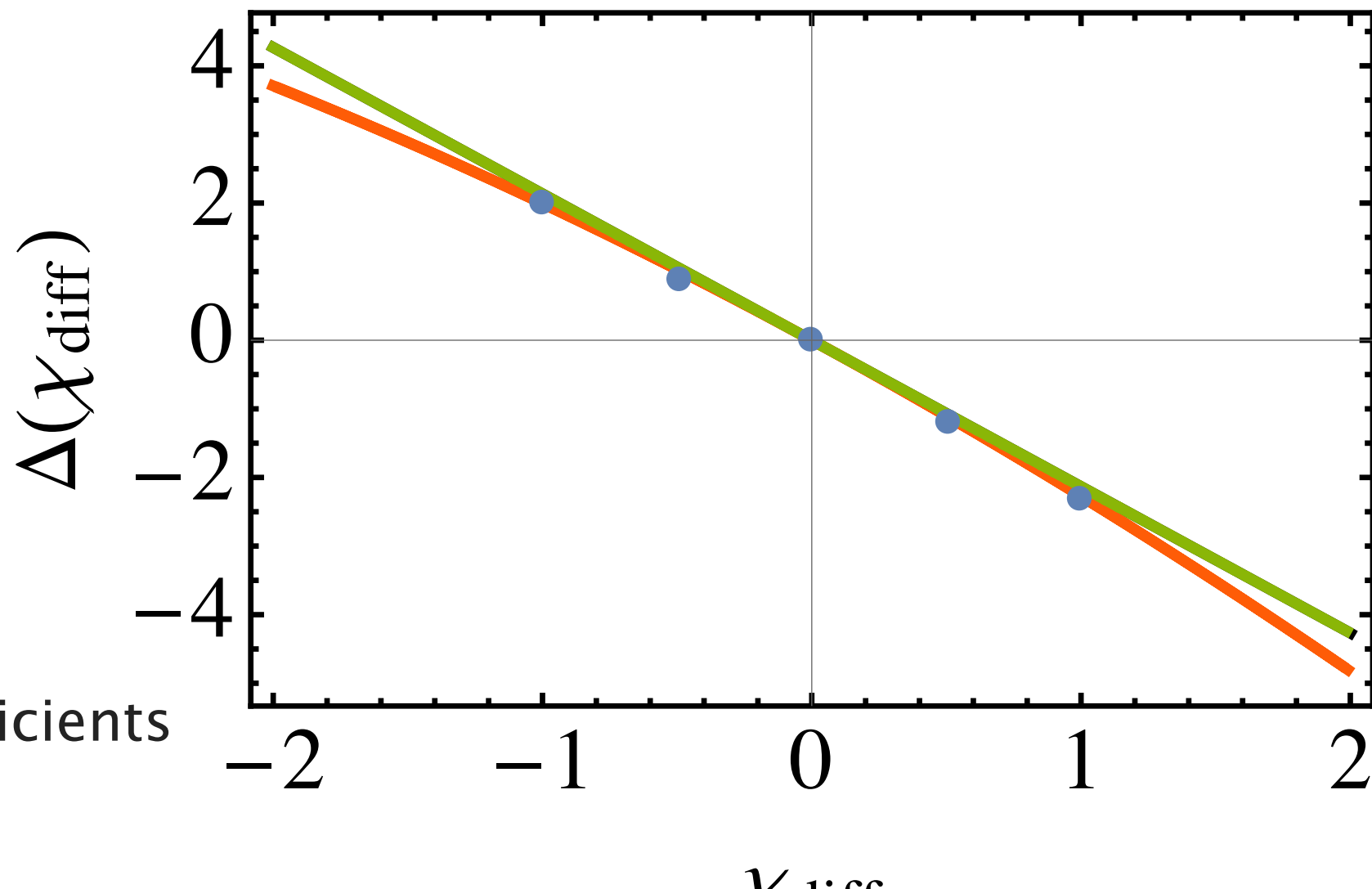
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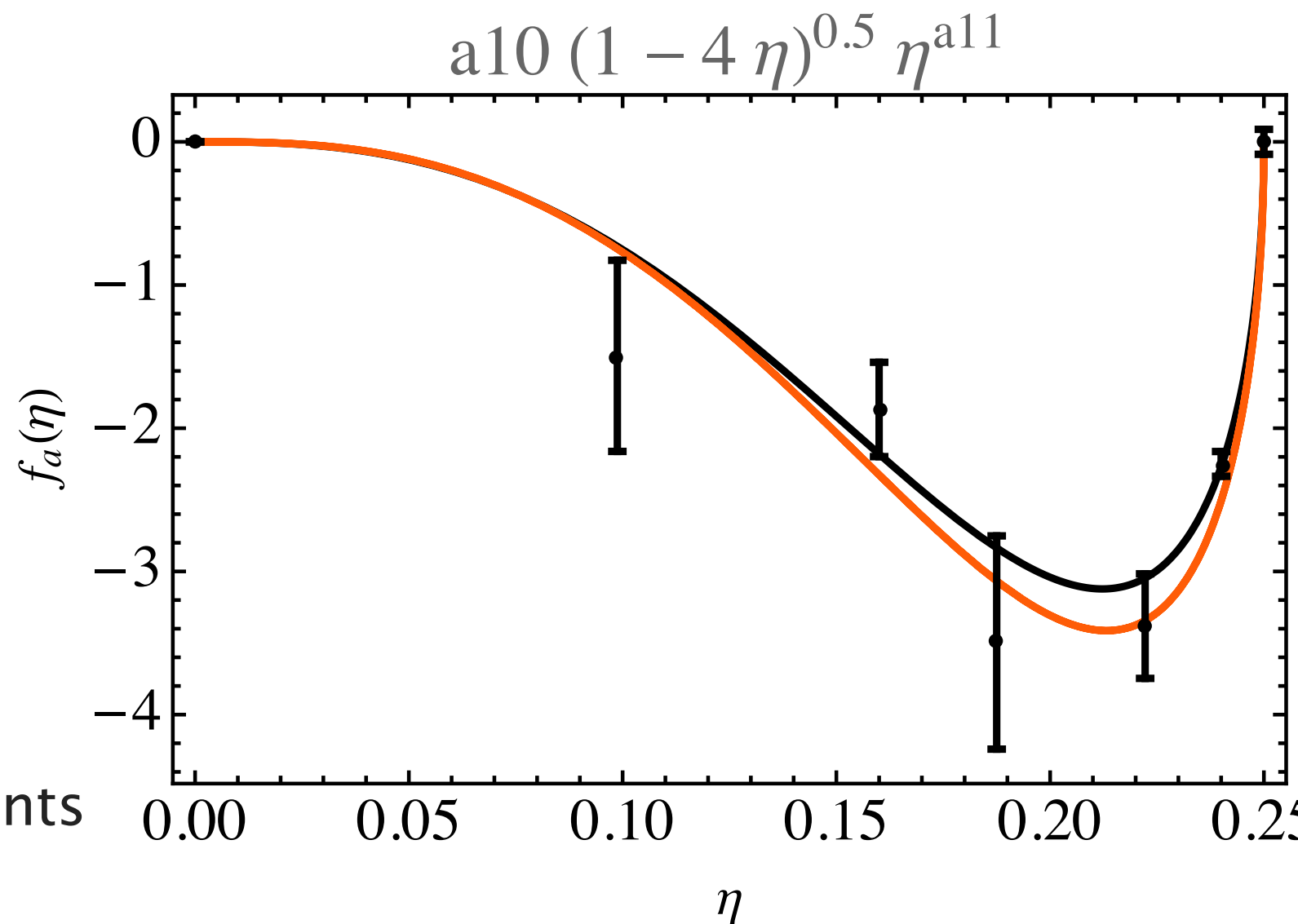
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# Summary

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- PhenomD: very accurate WFs in time & frequency domain.
  - Builds upon EOB inspiral description & detailed study of WF anatomy.
- PhenomD is modular, e.g. inspiral and MRD can be tuned from different waveform sets, variations of Phen\* models easy to generate.
  - Did not talk about modifications for modGR, eccentricity ...
- IMRPhenomP: precessing IMR model that is fast enough for general applications in Bayesian inference.
- In progress: unequal spin extension of PhenomD
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Interested in helping?

We have an open postdoc position!