Can chaos be observed in quantum gravity?

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based on:
PH, M. Kubalova and A. Tsobanjan, PRD **86** 065014 (2012);
B. Dittrich, PH, T. Koslowski and M. Nelson arXiv:1508.01947, and
arXiv:1602.03237

GR and 'observables'

General Relativity is a gauge theory

$\Rightarrow$ physical observables should be diffeomorphism invariant

canonically:

- Dirac observables as 'constants of motion' of constraints
- dynamics relationally $\Rightarrow$ 'evolving constants of motion' **[Wheeler 60's; Rovelli 90's; Dittrich '06,'07......]**

$\Rightarrow$ notoriously difficult to construct

often overlooked: even absent in presence of chaos

1. what then is observable?
2. consequences for QT?

Non-integrability and constraints [Dittrich, PH, Koslowski, Nelson '15; '16]

recall: non-integrable (unconstrained) systems:

- no global (smooth) constants of motion other than H exist

$\Rightarrow$ trajectories lie on $(2N-1)$-dim. energy surface

now: constrained system weakly non-integrable if:

$\nexists$ differentiable Dirac observables indep. of constraints

$\Rightarrow$ $\nexists$ reduced phase space

$\Rightarrow$ gauge invariant DoFs exist, but non-differentiable (or local)

$\Rightarrow$ no Poisson algebraic structure $\Rightarrow$ how to represent observables in QT?

difference:

unconstrained: do not need to solve dynamics

constrained: need to solve dynamics

to access physical DoFs

Why worry about this?

GR likely weakly non-integrable (N-body problem, Mixmaster, BKL,...)

$\Rightarrow$ (probably) $\nexists$ smooth Dirac observables and reduced phase space in full GR

$\Rightarrow$ what are repercussions for QG?

Toy model: free particles on a circle [Dittrich, PH, Koslowski, Nelson '15; '16]

Compactify free dynamics: $x_i + 1 \sim x_i$, $i = 1, 2 \Rightarrow$ conf. manf. $\mathcal{Q} \simeq T^2$

$$C = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} - E \approx 0$$

- solutions to EoMs

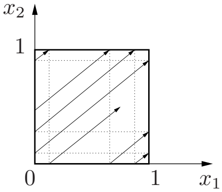
$$x_i(t) \quad = \quad \frac{p_i}{m_i} t + x_{i0} - n_i$$

$n_i := \lfloor \frac{p_i}{m_i} t + x_{i0} \rfloor$ winding number in x_i

if:

$\frac{m_2}{m_1} \frac{p_1}{p_2} \in \mathbb{Q}$: resonant torus, periodic orbits

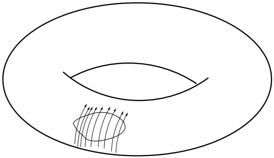
$\frac{m_2}{m_1} \frac{p_1}{p_2} \notin \mathbb{Q}$: non-resonant torus, ergodic orbits



Absence of sufficiently many Dirac observables [Dittrich, PH, Koslowski, Nelson '15; '16]

- momenta p_i are Dirac observables
- BUT: $O(p_i; x_1, x_2)$ const. on trajectories with $\partial_i O \neq 0$ discontinuous in x_i

  - ergodicity destroys integrability



- space of solutions not a manifold

$\Rightarrow$ no reduced phase space, no (sufficient) algebra of observables

- can still have gauge invariant 'observables', however, either
  1. global and discontinuous, e.g.

$$M = (x_1 + n_1)p_2/m_2 - (x_2 + n_2)p_1/m_1$$

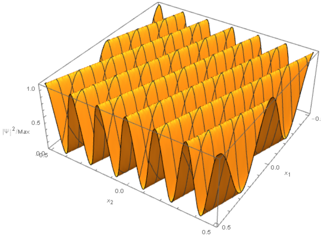
  2. local [Bojowald, PH, Tsobanjan '11a; '11b]

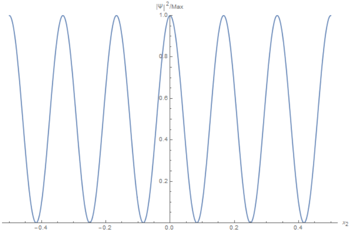
Standard Dirac quantization: no semiclassical limit [Dittrich, PH, Koslowski, Nelson '15; '16]

- $\mathcal{H}_{\rm kin} = L^2(T^2)$ basis:
  $\psi_{k_1,k_2}(x_1,x_2) = \exp(2\pi i k_1 x_1)\exp(2\pi i k_2 x_2), \qquad \vec{k} \in \mathbb{Z}^2$
- $\hat{p}_i \psi = -i\hbar\partial_i\psi$
- quantum constraint

$$k_1^2 + \frac{m_1}{m_2}k_2^2 = \frac{2m_1 E}{\hbar^2}$$

- for $m_1/m_2 \notin \mathbb{Q}$ $0 \leq \dim \mathcal{H}_{\rm phys} \leq 4$





NOT peaked on class. orbit for $m_1/m_2 \notin \mathbb{Q}$ width/separation ≈ 1

- physical transition amplitudes show no semiclassical behaviour

Polymer type quantization: discrete topology [Dittrich, PH, Koslowski, Nelson '15; '16]

- $\mathcal{H}_{\rm kin}$ given by (uncountable) basis

$$\psi_{x_1',x_2'}(x_1,x_2) = \delta_{x_1',x_1}\delta_{x_2',x_2}$$

- no momenta, but translations

$$(R_1^\mu\,\psi)(x_1,x_2) = \psi(x_1+\mu,x_2), \qquad (R_2^\mu\,\psi)(x_1,x_2) = \psi(x_1,x_2+\mu)$$

- constraint acquires continuous spectrum for $\mu \notin \mathbb{Q}$:

$$\{\frac{\hbar^2}{\mu^2}\,(2-\cos(2\pi\rho_1)-\cos(2\pi\rho_2)) - E|\rho_1,\rho_2\in[0,1)\}$$

$\Rightarrow$ upon superselec. get ∞-dim. separable $\mathcal{H}_{\rm phys}$ as L^2 over 'momentum' ρ

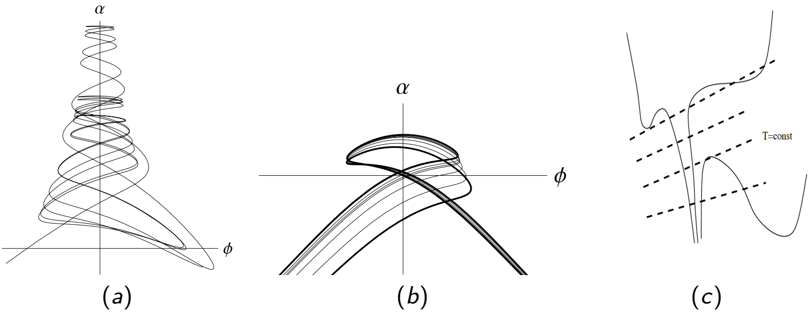
$\Rightarrow$ on this $\mathcal{H}_{\rm phys}$ have sufficiently many observables

$$\hat{M} := \frac{i}{2\pi}\left(\sin(2\pi\rho_2)\frac{\partial}{\partial\rho_1} - \sin(2\pi\rho_1)\frac{\partial}{\partial\rho_2}\right) \qquad \text{('angular mom.')}$$

$$[\hat{M}, e^{2\pi i\rho_1}] = -e^{2\pi i\rho_1}\,\sin(2\pi\rho_2)$$

$\Rightarrow$ features good semiclassical transition amplitudes

Closed FRW with (min. coupled) massive scalar [PH, Kubalova, Tsobanjan, '12]



(a) benign solution, (b) defocussing of nearby trajectories in turning region, (c) devoid of global clocks

- Ham. constraint $C = p_\phi^2 - p_\alpha^2 - e^{4\alpha} + m^2\,\phi^2\,e^{6\alpha}$
- model chaotic and non-integrable [Page '84, Cornish, Shellard '98; Belinsky, Khalatnikov, Grishchuk, Zeldovich '85]
- in region of max. expansion α_{max} (chaotic scattering):
  1. breakdown of semiclassicality
     [also indep. observed in Kiefer '88]
  2. relational observables only transient

     $\Rightarrow$ relational evolution breaks down

Conclusions

- Chaos precludes smooth Dirac observables

$\Rightarrow$ probably no smooth Dirac observables and red. phase space for full GR

- serious problem for ‘standard’ constraint quantization

what do we do?

- always $\exists$ generalized discontinuous ‘observables’

$\Rightarrow$ adapt method of quantization, refine topology until sufficiently many observables continuous

$\Rightarrow$ here: polymer quantization overcomes troubles of ‘standard’ quantization!

further reading: arXiv:1602.03237, 1508.01947, 1111.5193