Phases of Gravity with Anisotropic Scaling

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Work in progress with Petr Hořava and Charles Melby-Thompson

(2+1)-d Projectable Hořava-Lifshitz Gravity

ADM fields: N (Lapse), N_i (Shift), g_{ij} (spatial metric).

Projectability: $N(t, \mathbf{x}) = N(t)$.

The action:

$$S = \frac{1}{2\kappa^2} \int dt\, d^2x\, N\sqrt{g}\Big(K_{ij}K^{ij} - \lambda K^2 - \frac{\alpha}{2}R^2 + \beta R - 2\gamma\Big)$$

Extrinsic curvature: $K_{ij} = \frac{1}{2N}(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i)$.

$z = 2$ dimensions: $[\kappa] = [\lambda] = [\alpha] = 0$, $[\beta] = 2$, $[\gamma] = 4$.

FLRW Solutions

Ansatz: $N = 1$, $N_i = 0$, $g_{ij} = f(t)\,\hat{g}_{ij}$, where $\hat{R}_{ij} = k\hat{g}_{ij}$.

$k = 1$ for S^2 spatial slices (CDT).

Study solutions as functions of $\alpha, \beta, \gamma, \lambda, \kappa^2$ and k.

$|\alpha| = 1$ near $z = 2$ fixed point.

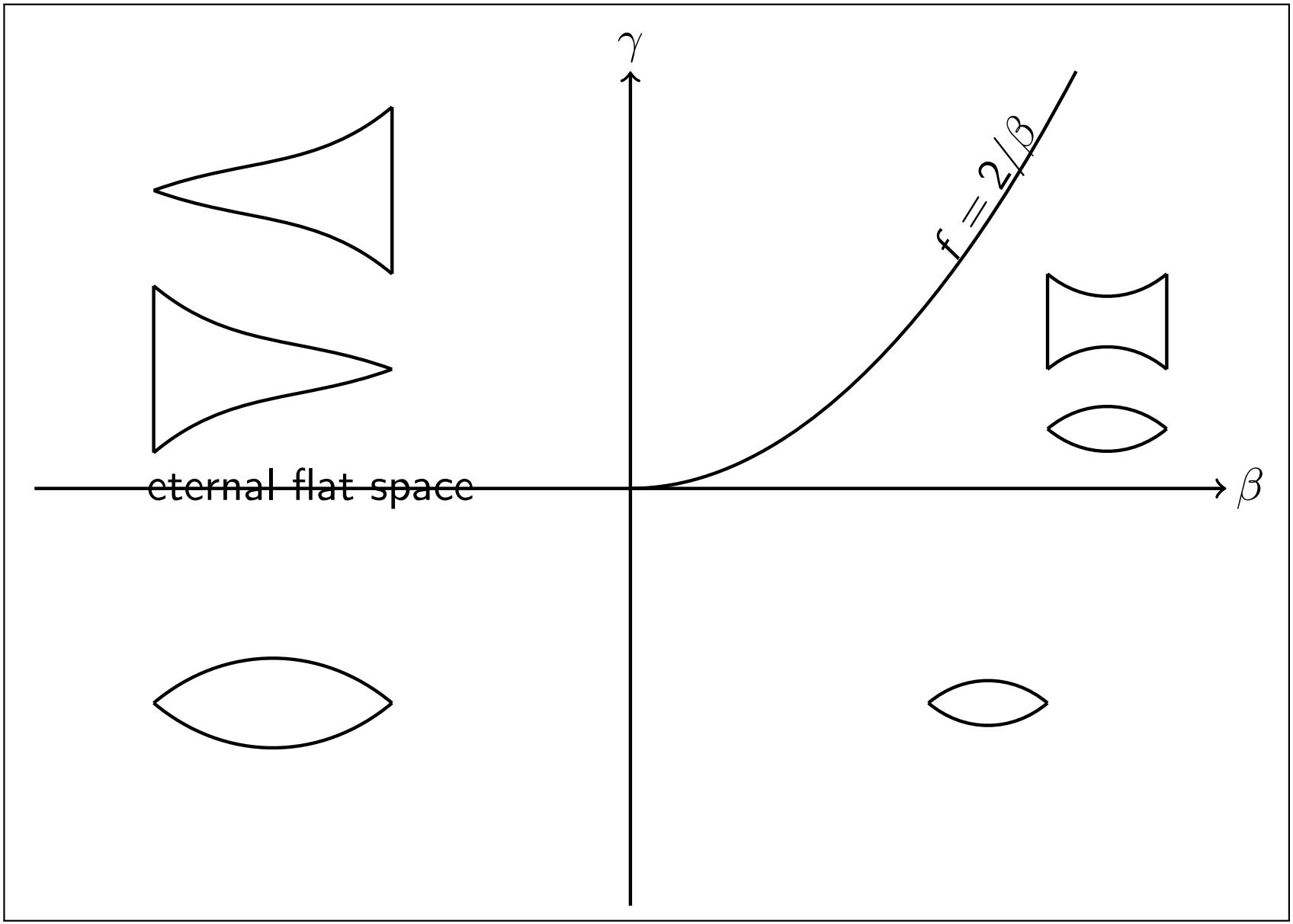
Scalar graviton is a ghost if $\frac{1}{2} < \lambda < 1 \Longrightarrow$ excise or $\kappa^2 < 0$.

The Case of $\lambda = \frac{1}{2}$

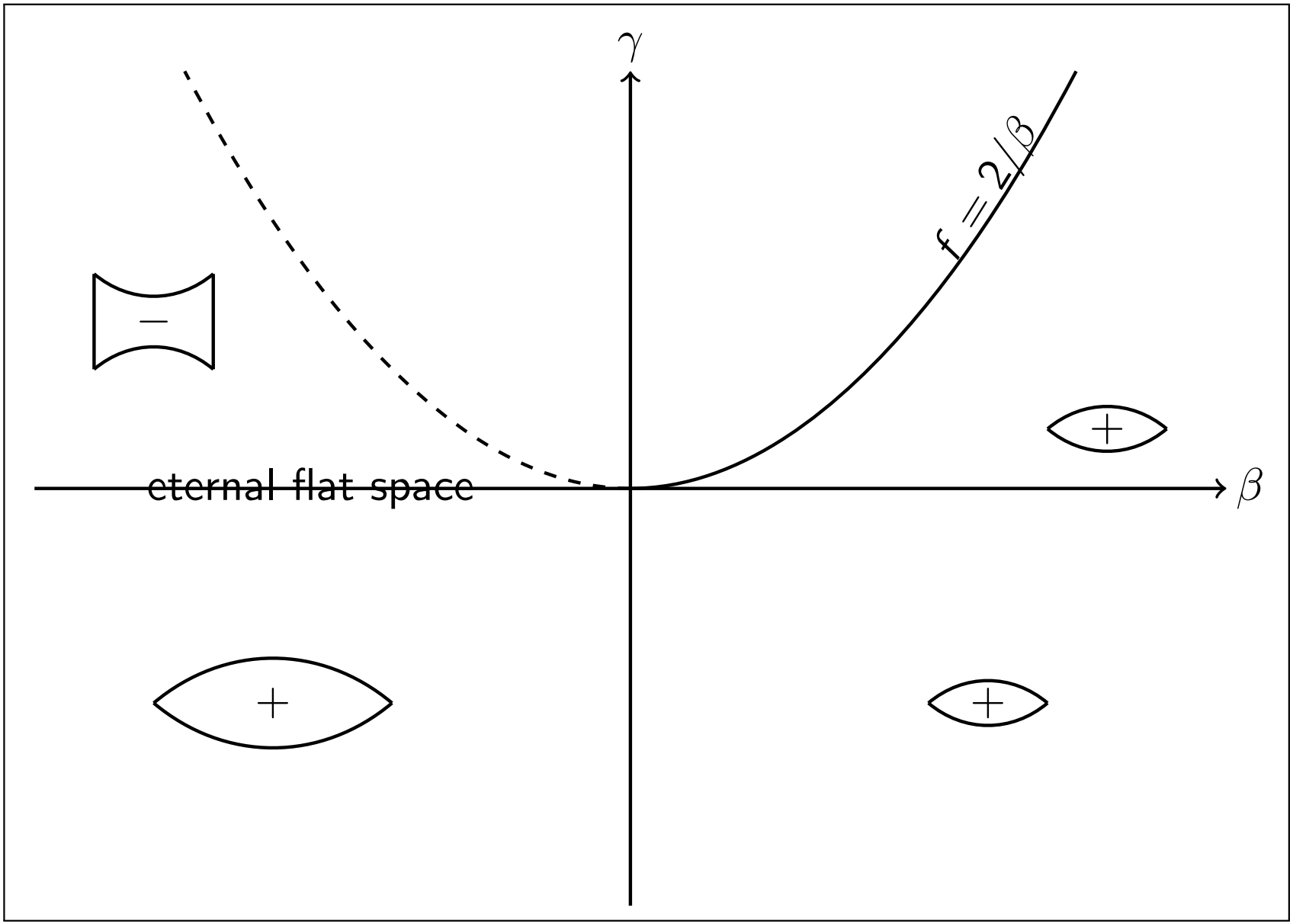
$z = 2$ Weyl symmetry in $\alpha, \beta, \gamma \to 0$ limit.

$f = 2\alpha k/\beta$ (constant) as long as this is positive and $\beta^2 = 4\alpha\gamma$.

The Case of $\lambda > \frac{1}{2}$, $\alpha > 0$, $k = 1$



Decompactification at Negative β

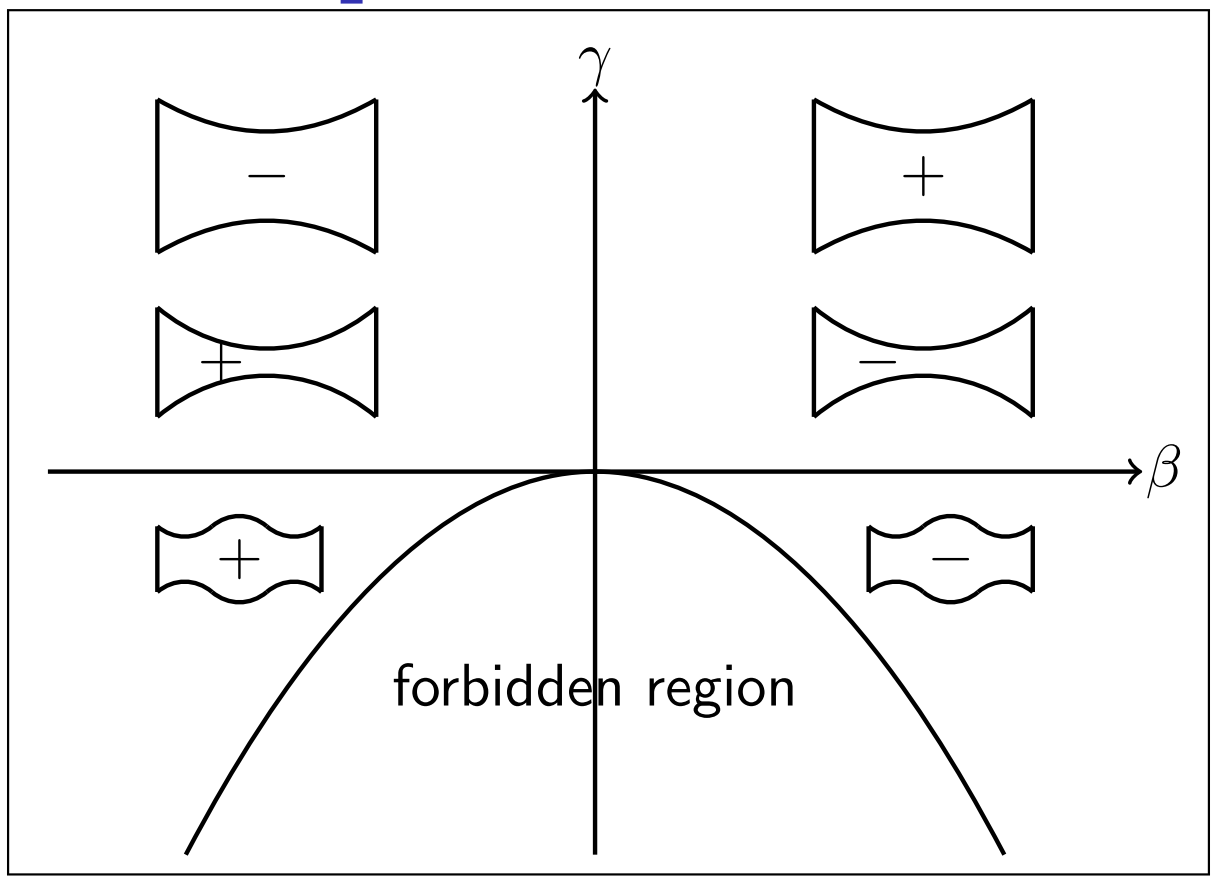


Forbidden Regions

The Case of $\lambda < \frac{1}{2}$ and $\alpha > 0$

Diagram is vertically flipped. No solution inside the parabola!

The Case of $\lambda > \frac{1}{2}$ and $\alpha < 0$

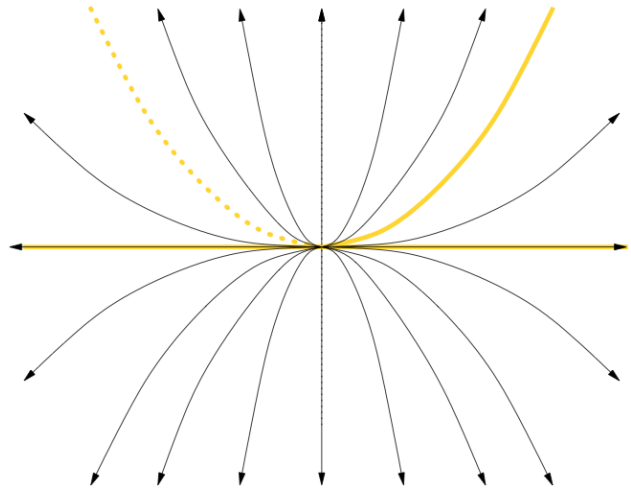


Thoughts on the Forbidden Region

1. Do not impose the N eom (it is not imposed in CDT). Extremize energy. However, energy is unbounded from below!
2. Keep N variable. Perhaps $N \to 0$ in the forbidden region. Topological phase of time?
3. $\alpha < 0 \Longrightarrow$ add stabilizing higher terms like $\nabla_i R \nabla^i R$. Reduced spatial symmetries?
4. Add non-projectable terms like $N^{-2} \Box N$.

Comments on the Phase Diagram and RG Flows

1. Transition lines are mutually tangent at the tricritical point.
2. Mean field RG flows are parabolic.



3. Fluctuations $\Longrightarrow$ anomalous dimensions $\Longrightarrow$ more generic.

Thoughts on the Comparison to $(3+1)$-d CDT

1. Lifshitz renormalizability: $z = 3$ Gaussian fixed point.
   Asymptotic safety: $z = 1$ strongly coupled fixed point.
   Hybrid: Nontrivial fixed point with z near 2?
2. If $(2+1)$-d theory is asymptotically free, could it be a Wilson-Fisher fixed point?

Thanks for your attention.

Tak for din opmærksomhed.