Transverse deformations of extremal horizons

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GR21, Mathematical Relativity and Classical Gravitation Session,
NYC, 13 July 2016

Extremal black holes

- Classify $D \geq 4$ stationary black hole solutions to Einstein eqs. **Extremal**: surface gravity $\kappa = 0$. No Hawking temperature.

- No-hair theorem only recently extended to extremal Kerr black hole. 'Boundary' condition at horizon differs.
  [Meinel et al '08; Amsel et al '09; Figueras, JL '09; Chrusciel, Nguyen '10]

- **Near-horizon geometry** solves Einstein eq: $\mathrm{AdS}_2 \times S^2$... Classify these! [Reall '02, ... Kunduri, JL - Living Reviews '13]

  But which correspond to black holes?

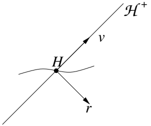
- Existence and uniqueness of extremal black holes with given near-horizon geometry *not* guaranteed.

Extremal horizons and near-horizon geometry

- Smooth Killing horizon $\mathcal{H}^+$ with *compact* cross-section H:

  Normal Killing field n; extremality $\mathrm{d}n^2|_{\mathcal{H}^+} = 0$

  Gaussian null coordinates [Moncrief, Isenberg '83]
  $n = \frac{\partial}{\partial v}$, $\ell = \frac{\partial}{\partial r}$ null geodesic, $\mathcal{H}^+ = \{r = 0\}$



$$\boxed{g = 2\,\mathrm{d}v\left(\mathrm{d}r + r\,h_a(r,x)\mathrm{d}x^a + \tfrac{1}{2}r^2F(r,x)\mathrm{d}v\right) + \gamma_{ab}(r,x)\mathrm{d}x^a\mathrm{d}x^b}$$

- **Near-horizon geometry** $\equiv \mathcal{H}^+$ *intrinsic* data $(\gamma_{ab}, h_a, F)_{r=0}$

  Define $g_\epsilon = \phi_\epsilon^* g$ by scaling $\phi_\epsilon : (v, r, x^a) \mapsto (v/\epsilon, \epsilon r, x^a)$

  Near-horizon limit: $g_\epsilon \to g_{\mathrm{NH}}$ as $\epsilon \to 0$ exists where $\gamma_{ab} \to \gamma_{ab}|_{r=0}$ etc

  $\mathrm{Ric}(g) = 0 \implies \mathrm{Ric}(g_{\mathrm{NH}}) = 0 \iff R_{ab} = \frac{1}{2}h_a h_b - D_{(a}h_{b)}$ on H

The inverse problem [Li, JL '15]

- Given $(\gamma_{ab}, h_a, F)_{r=0}$ determine $(\gamma_{ab}, h_a, F)_{r>0}$. Too hard!
  Simpler: determine 1^{st} order $\gamma^{(1)}_{ab} = \partial_r \gamma_{ab}|_{r=0}$ etc
- Captures extrinsic curvatures of H: $\chi^{(\ell)}_{ab} = \frac{1}{2}\gamma^{(1)}_{ab}$ $(\chi^{(n)}_{ab} = 0)$.
  Encoded by **transverse deformation** $g^{(1)} = (\mathrm{d}g_\epsilon/\mathrm{d}\epsilon)|_{\epsilon=0}$

  Diffeos $g^{(1)} \to g^{(1)} + \mathcal{L}_\xi g_{\mathrm{NH}}$, $\xi = f(x)\partial_v + \dots$ [cf. supertranslations]

$$\gamma^{(1)}_{ab} \to \gamma^{(1)}_{ab} + D_a D_b f - h_{(a} D_{b)} f$$

  Linearised Einstein eq in 'background' g_{NH} reduces to elliptic eq on H:

$$\boxed{\Delta_L \gamma^{(1)}_{ab} + \frac{1}{2} D_a D_b \gamma^{(1)} + (h\, D\gamma^{(1)})_{ab} + (h^2 + Dh)(\gamma^{(1)})_{ab} = 0}$$

Theorem. The moduli space of transverse deformations of an extremal horizon with compact H is finite dimensional (up to gauge).

Marginally trapped surfaces (MTS) and extremality [Li, JL '15]

- Require H is MTS: $\theta_n = 0$ and $\theta_\ell = \frac{1}{2}\gamma^{(1)} > 0$
  1^{st} order: only $\Theta \equiv \int_H \theta_\ell > 0$ is gauge inv (after $\ell \to \Gamma\ell$)

- Non-extremal BH: 'stable' $\mathcal{L}_\ell \theta_n|_H > 0$ so $\theta_n < 0$ just in BH.
  Extremal BH no TS! $\mathcal{L}_\ell \theta_n|_H = 0$ [Booth, Fairhurst '08; Mars '12]

- **Stability** of extremal MTS: $\boxed{\mathcal{L}_\ell \mathcal{L}_\ell \theta_n|_H > 0}$

  $\mathcal{L}_\ell \mathcal{L}_\ell \theta_n|_H = -A\gamma^{(1)} + D \cdot v$, where A is fixed by horizon data.

  $SO(2,1) \times U(1)$ near-horizon symmetry: Einstein eq: $A < 0$ const [Kunduri, JL, Reall '07]

If H is compact extremal MTS then $\int_H \mathcal{L}_\ell \mathcal{L}_\ell \theta_n = -A\Theta > 0$

Four dimensional deformations

- Near-horizon uniqueness [Lewandowski, Pawlowski '03; Kunduri, JL '08]: any smooth, *axisymmetric*, extremal vacuum horizon is Kerr:

$$\gamma_{ab}\mathrm{d}x^a\mathrm{d}x^b = a^2\frac{1+x^2}{1-x^2}\mathrm{d}x^2 + 4a^2\frac{1-x^2}{1+x^2}\mathrm{d}\phi^2$$

- *Axisymmetric* deformations: reduce to a 2^{nd} order ODE for gauge inv $X = 4x\gamma^{(1)}_{x\phi} - (1+x^2)\gamma^{(1)}_{\phi\phi}$. Smooth $\implies$ $X \equiv 0$.

Theorem [Local uniqueness of extremal Kerr horizon] [Li, JL '14]

Any smooth marginally trapped transverse deformation of extremal Kerr horizon is gauge equiv to 1^{st}-order Kerr black hole data.

Independent to no-hair theorem: no global assumptions (e.g. asymptotic flatness...)

Five dimensional deformations [Li, JL '14]

- $\mathbb{R}_t \times U(1)^2$ symmetry. Myers-Perry black hole $J_1 = J_2$ and KK black hole $J = 0$: same near-horizon geometry $H = S^3$. [Kunduri, JL '08]

- $U(1)^2$-deformations: equivalent to 5 coupled ODEs for 5 gauge invariant variables. Can be fully integrated!

  General smooth $\gamma^{(1)}_{ab}$ is 3-parameter family; known black holes 0-parameter subset. What do other solutions correspond to?

- Black hole + bubble $H_2(\text{DOC}) \neq 0$? 'Black lens' $H = L(p, q)$? Supersymmetric black holes of this kind exist! [Kunduri, JL '14] (see String Theory Session later)

Summary and Future directions

- Space of infinitesimal **transverse deformations** of extremal horizons is **finite dimensional**.

  $D = 4$ **uniqueness theorem**: vacuum w/ axisymmetry.

  $D = 5$ **classification** w/ biaxisymmetry. Solution more general than known black holes... new vacuum black holes?

- Λ: Black holes in AdS with single corotating Killing field?

  Matter fields: Einstein-Maxwell uniqueness?

  Higher order transverse deformations?
  Constructive extremal black hole uniqueness proofs?