CONSERVED CHARGES FOR INTEGRABLE QFTS IN INFLATION

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INTRODUCTION AND MOTIVATION

- Many studies of quantum effects in cosmology rely on perturbation theory (Cf. talks in B4, C4!)

Solving exact problem approximately

- Alternative approach: study integrable models

Solving approximate problem exactly

Constructing integrable QFTs on cosmological backgrounds stands to give insight both to quantum cosmology and QFT.

THIS TALK:

- Examine a class of QFTs known to be integrable on Minkowski space.
- Show that on dS background these thys retain class of charges with powerful Ward identities.
- Suggests certain features of dS thy might be integrable (asymptotic correlations functions? dS S-matrix? dS corrections to effective mass?)

A CLASS OF INTEGRABLE MODELS

- Consider nonlinear sigma model on dS_2 w/ target space a symmetric space (classical group G)

$$S = \frac{1}{2a^2} \int d^2x \sqrt{-\gamma}\, \mathrm{tr}[\nabla_\mu \phi^{-1} \nabla^\mu \phi] \qquad \Box\phi - (\nabla_\mu \phi)\phi^{-1}\nabla^\mu \phi = 0$$

Fundamental field $\phi \in G, \gamma_{\mu\nu}$ is spacetime metric. E.g., $G = SU(N)$

- Theory has many symmetries!

- Noether currents of L and R global group transformations

$$\phi(x) \to U_L \phi(x) U_R^{-1}, \quad U_L \in SU(N)_L,\, U_R \in SU(N)_R$$

$$j_\mu := j_\mu^L = \frac{1}{a^2}\nabla_\mu \phi \phi^{-1}, \quad j_\mu^R = -\frac{1}{a^2}\phi^{-1}\nabla_\mu \phi$$

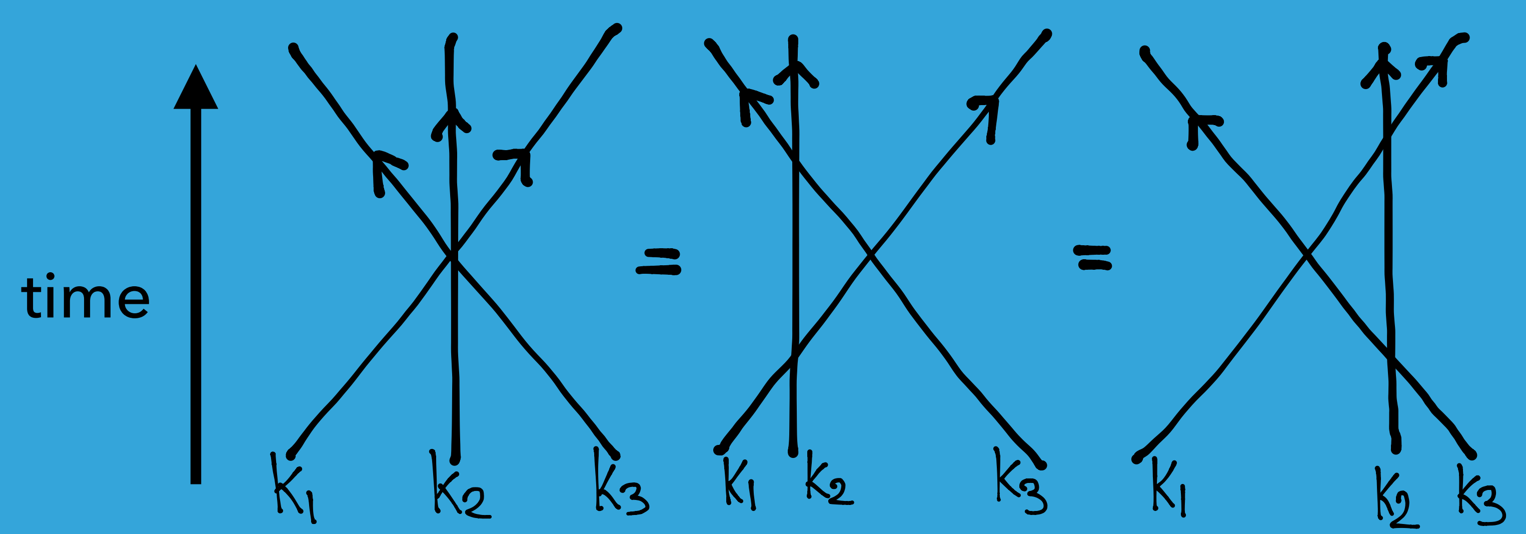
$$S = \frac{a^2}{2}\int d^2x \sqrt{-\gamma}\, \mathrm{tr}[j_\mu j^\mu] \qquad \nabla^\mu j_\mu = 0$$

- "Flatness condition"

$$\nabla_\mu j_\nu - \nabla_\nu j_\mu + a^2 [j_\mu, j_\nu] = 0.$$

CHARGES AND THE MINKOWSKI S-MATRIX

- One way to obtain S-matrix for our thys is by exploiting Ward identities associated with (unusual) conserved charges.
- Local higher-spin charges relates N-to-N scattering to products of 2-to-2 scattering.



Yang-Baxter Eqn.

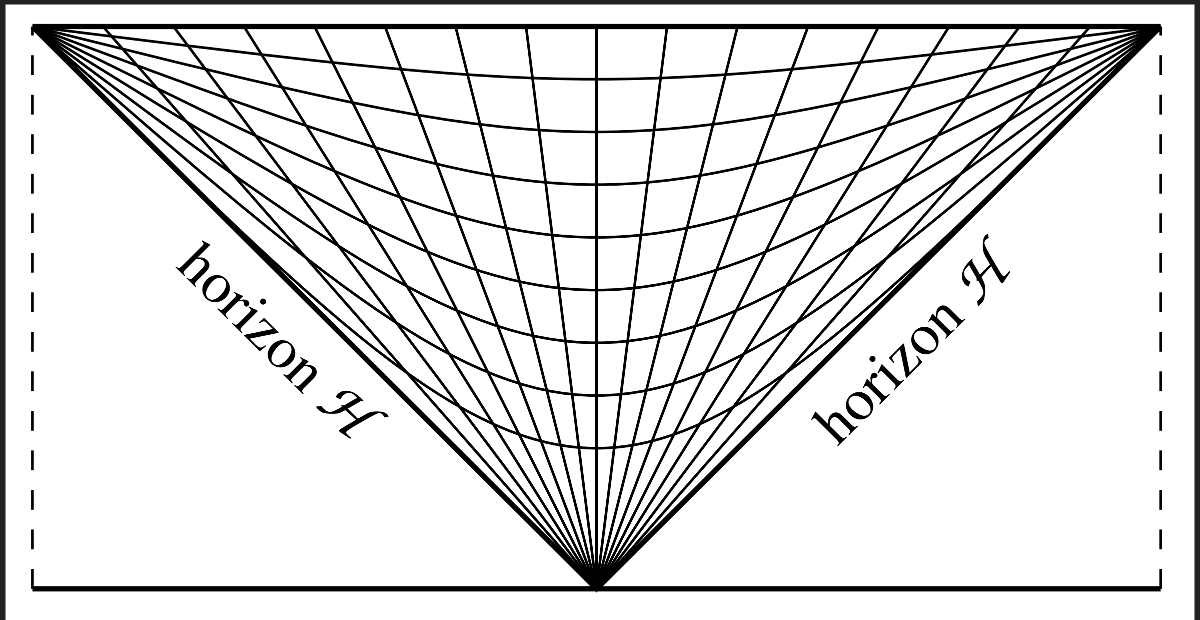
$$S(k_1,k_2,k_3)=S(k_1,k_2)S(k_1,k_3)S(k_2,k_3)=S(k_2,k_3)S(k_1,k_3)S(k_1,k_2)$$

- "First non-local charge" $Q^{(1)}$ yields relations between 2-particle states, determines 2-to-2 scattering.

[Luscher, Parke, Shankar Witten, Abdalla et. al., …]

DE SITTER SPACETIME

Poincaré (Cosmological) chart:



$$ds^2 = \frac{\ell^2}{\eta^2}\left(-d\eta^2 + dy^2\right)$$

$$\eta \in (-\infty, 0),\ \ y \in \mathbb{R}$$

- de Sitter radius $\ell = 1/H$
- dS_2 Killing vectors:
  - translation ∂_y
  - dilation $\eta\partial_\eta + y\partial_y$
  - SCT $2\eta y\partial_\eta + (\eta^2 + y^2)\partial_y$
- Flatspace limit: $\ell \to \infty$, t fixed

$$\eta = -\ell e^{-t/\ell}, \quad t = -\ell \ln\left(-\frac{\eta}{\ell}\right),$$

$$ds^2 = -dt^2 + e^{2t/\ell}dy^2, \quad t, y \in \mathbb{R}$$

LOCAL HIGHER SPIN CHARGES

$$Q^{(s)} = \int d\Sigma\, n^\mu J_{\mu\nu_1\ldots\nu_s} K^{\nu_1\ldots\nu_s}$$

- HS charges transform as tensor component under dS transformations. They enlarge isometry algebra in an interesting way.
- Upon quantization anomalies can spoil conservation laws.
- Anomaly counting arguments suffice to prove existence of currents in quantum theory.
- **Conclusion:** on dS at least one spin-4 current survives quantization for all classical groups.

[Goldschmidt Witten, Evans et. al]

Ex: for G = SUN(N)

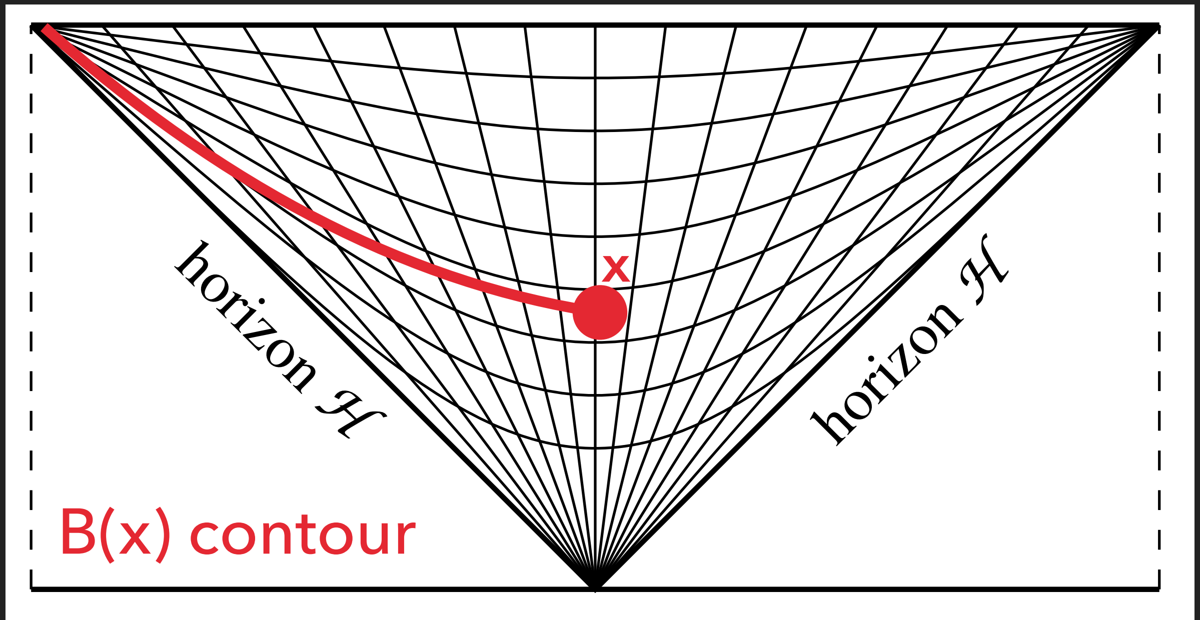
$$\begin{aligned} T_{\mu\nu} &= \mathrm{tr}\left[j_\mu j_\nu - \frac{1}{2}\gamma_{\mu\nu} j_\lambda j^\lambda\right] \\ J^{(3)}_{\mu\nu\lambda} &= \mathrm{tr}\left[j_{(\mu} j_\nu j_{\lambda)} - \frac{3}{4}\gamma_{(\mu\nu} j_{\lambda)} j_\sigma j^\sigma\right], \\ J^{(4a)}_{\mu\nu\lambda\sigma} &= \mathrm{tr}\left[j_{(\mu} j_\nu j_\lambda j_{\sigma)}\right] - \mathrm{traces}, \\ J^{(4b)}_{\mu\nu\lambda\sigma} &= \mathrm{tr}\left[j_{(\mu} j_\nu\right] \mathrm{tr}\left[j_\lambda j_{\sigma)}\right] - \mathrm{traces}. \end{aligned}$$

$$\nabla^\mu J^{(s)}_{\mu\nu_1\ldots\nu_s} = \sum_i c_i a^{(i)}_{\nu_1\ldots\nu_s}$$

Ex: for s=3:

$$\begin{aligned} a^{(1)}_{\mu\nu} &= \gamma_{\mu\nu}\nabla_\lambda \mathrm{tr}\left[j^\lambda j^2\right], \\ a^{(2)}_{\mu\nu} &= \nabla_{(\mu}\mathrm{tr}\left[j_{\nu)} j^2\right] - \frac{1}{2} a_{1\mu\nu}, \\ b^{(1)}_{\mu\nu} &= \nabla^\lambda\left(\gamma_{(\mu\nu}\mathrm{tr}\left[j_{\lambda)} j^2\right]\right). \end{aligned}$$

NON-LOCAL CHARGES



- "Flatness condition" assures dB=*j may be integrated

$$B(\eta, y) = -\int_{-\infty}^{y} d\bar{y}\, j_0(\eta, \bar{y})$$

- B(x) has non-trivial braid relations with with field

$$[B^c(\eta, y_1), \phi(\eta, y_2)] = \begin{cases} 0 & \text{if} \quad y_1 \leq y_2 \\ Q^a\left(\phi(x_2)\right) & \text{if} \quad y_1 > y_2 \end{cases} \qquad Q^a = \int dy\, j_0^a(\eta, y)$$

- Using B(x) may construct non-local current:

$$j_\mu^{(1)}(x) = (*j)_\mu(x) + \frac{1}{2}\left[j_\mu(x), B(x)\right]$$

$$Q^{(1)a} = -\int_{-\infty}^{+\infty} dy\, j_1^a(\eta, y) + \frac{i}{2} f^{abc} \int_{-\infty}^{+\infty} dy\, j_0^b(\eta, y) B^c(\eta, y)$$

non-trivial co-product

$$Q^{(1)a}\left(\phi(x_1)\phi(x_2)\right) = Q^{(1)a}\left(\phi(x_1)\right)\phi(x_2) + \phi(x_1) Q^{(1)a}\left(\phi(x_2)\right) - \frac{1}{2} f^{abc} Q^b\left(\phi(x_1)\right) Q^c\left(\phi(x_2)\right)$$

[Bernard, Brezin et. al, Abdall et. al.]

NON-LOCAL CHARGES

$$Q^{(1)a} = -\int_{-\infty}^{+\infty} dy\, j_1^a(\eta,y) - \frac{i}{2} f^{abc} \int_{-\infty}^{+\infty} dy \int_{-\infty}^{y} d\overline{y}\, j_0^b(\eta,y) j_0^c(\eta,\overline{y})$$

- Composite operator in 2nd term must be renormalized. Use OPE:

$$f^{abc} j^b_{\mu_1}(x_1) j^c_{\nu_2}(x_2) \sim C^{\lambda_1}_{\mu_1\nu_2}(X_{12}) j^a_{\lambda_1}(x_1) + D^{\alpha_1\beta_1}_{\mu_1\nu_2}(X_{12}) \nabla_{\alpha_1} j^a_{\beta_1}(x_1) + \ldots$$

divergent terms: $\quad f^{abc} j_0^b(\eta,y_1) j_0^c(\eta,y_2) \sim \frac{1}{\Delta y} j_1^a(\eta,y_1) + O(\ln \Delta y/L)$

- Conclusion: $Q^{(1)}$ survives quantization, renormalized charge is conserved. This requires finite, dS counterterms.

$$Q^{(1)}_{\text{ren}} = \lim_{\delta\to 0} \left[-\left(1 + \ln\delta/L + \ln(\eta/\ell)\right) \int_{-\infty}^{+\infty} dy\, j_1^a(\eta,y) - \frac{i}{2} f^{abc} \int_{-\infty}^{+\infty} dy \int_{-\infty}^{y-\delta} d\overline{y}\, j_0^b(\eta,y) j_0^c(\eta,\overline{y}) \right]$$

- Algebra w/ dS charges:

$$\left[P, Q^{(1)a}\right] = 0, \ \left[D, Q^{(1)a}\right] = 0, \ \left[K, Q^{(1)a}\right] = \ell \frac{C_2(G)}{2\pi} \int_{-\infty}^{+\infty} dy\, j_0^a(\eta,y)$$

CONCLUSION

SUMMARY:

- Examined non-linear sigma model w/ classical group target spaces on dS background.
- Shown that certain local higher-spin charges, as well as the “first” non-local charge, are preserved in dS quantum thy.
- These charges are members of large families of charges; our results imply some of these charges also survive quantization.

BIG QUESTION:

- In Minkowski, Ward identities associated with these charges are sufficient to solve for S-matrix.
- In dS, are the Ward identities strong enough to determine asymptotic correlation functions? (Late-time correlators, correlations between future and past infinity)