Quantum limits of laser interferometric gravitational-wave detectors

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Reporting joint research by:

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What is this about?

1. Basics about quantum noise

Why it is relevant and what it is

2. Standard Quantum Limit

Different approaches to surpassing it

3. Fundamental Quantum Limit / Energetic Quantum Limit / Quantum Cramér-Rao Bound

$$S_{hh}(\Omega) \geq \frac{\hbar^2 c^2}{4L_{\rm arm}^2 S_{PP}(\Omega)}$$

General condition for achieving it

4. Approaches to enhancing optical power fluctuation

“No” limit in the lossless case (optical loss issue is under study)

Outline

❖ Standard Quantum Limit

- Some introduction to quantum noise
- Different approaches for surpassing Standard Quantum Limit

❖ Fundamental Quantum Limit

- Gravitational-wave detection as quantum parameter estimation
- Condition for achieving fundamental quantum limit

❖ Approaches to Lowering Fundamental Limit

- External squeezing and internal ponderomotive squeezing
- General coherent optical feedback

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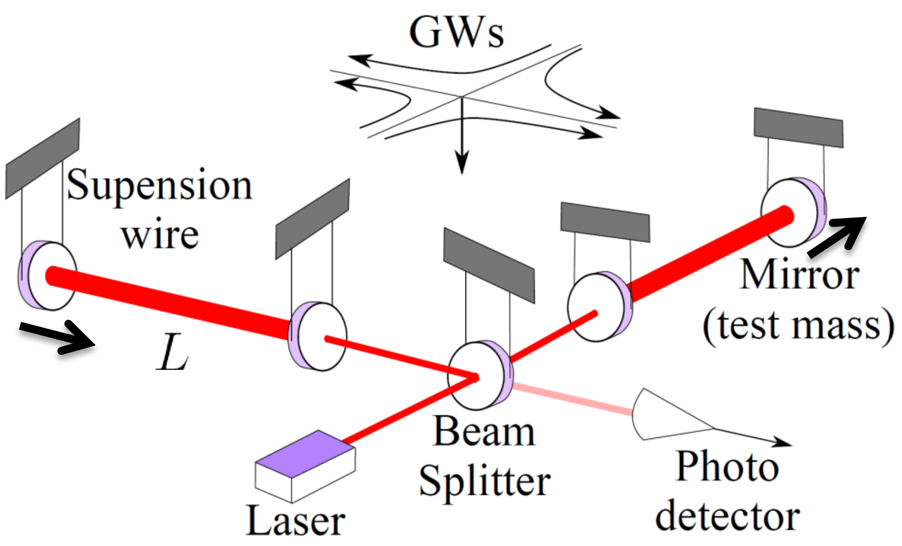
Why quantum?

Laser interferometric gravitational-wave (GW) detector:



Displacement Sensitivity of Advanced LIGO:

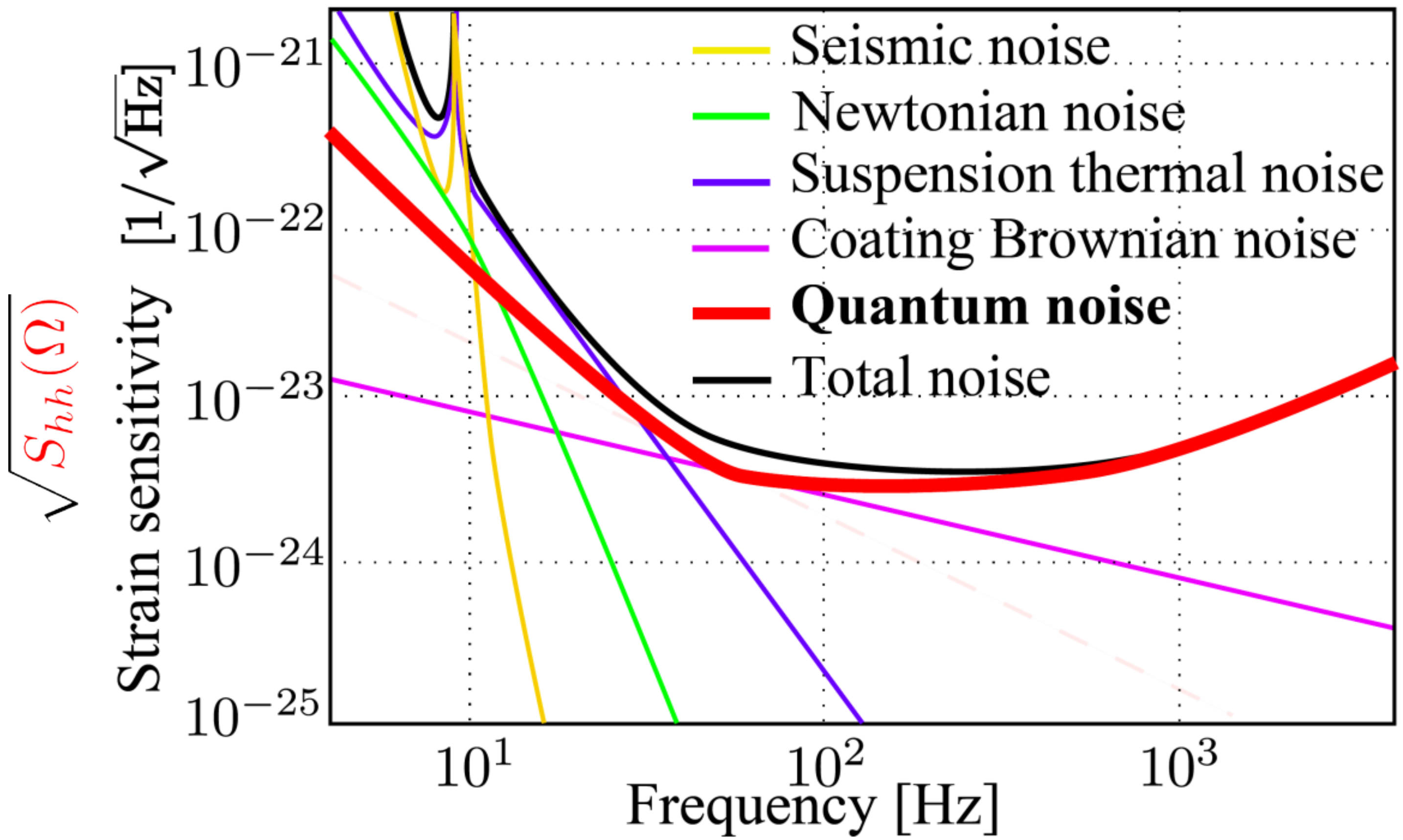
$$\Delta x \sim \sqrt{S_{hh}(\Omega) L_{\rm arm} \Delta\Omega}|_{100{\rm Hz}} \sim 10^{-19}\,{\rm m}$$

de Broglie wavelength of **40kg test mass**:

$$\lambda_{\rm d} \sim \sqrt{\hbar/(2M\,\Omega)}|_{100{\rm Hz}} \sim 10^{-19}\,{\rm m}$$

Quantum limited Advanced LIGO

Advanced LIGO design sensitivity curve:

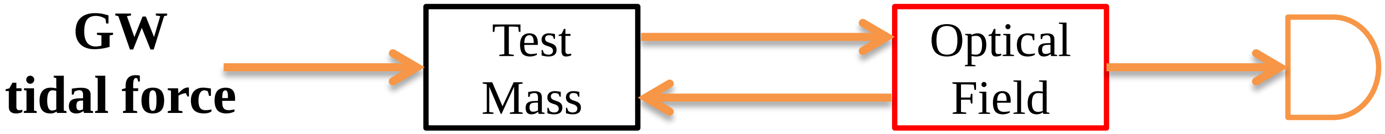


For known GW waveform: $h_{\mathrm{GW}}(\Omega)$

$$\mathrm{SNR} = \int \frac{|h_{\mathrm{GW}}(\Omega)|^2}{S_{hh}(\Omega)} \frac{d\Omega}{2\pi}$$

noise spectrum $S_{hh}(\Omega)$

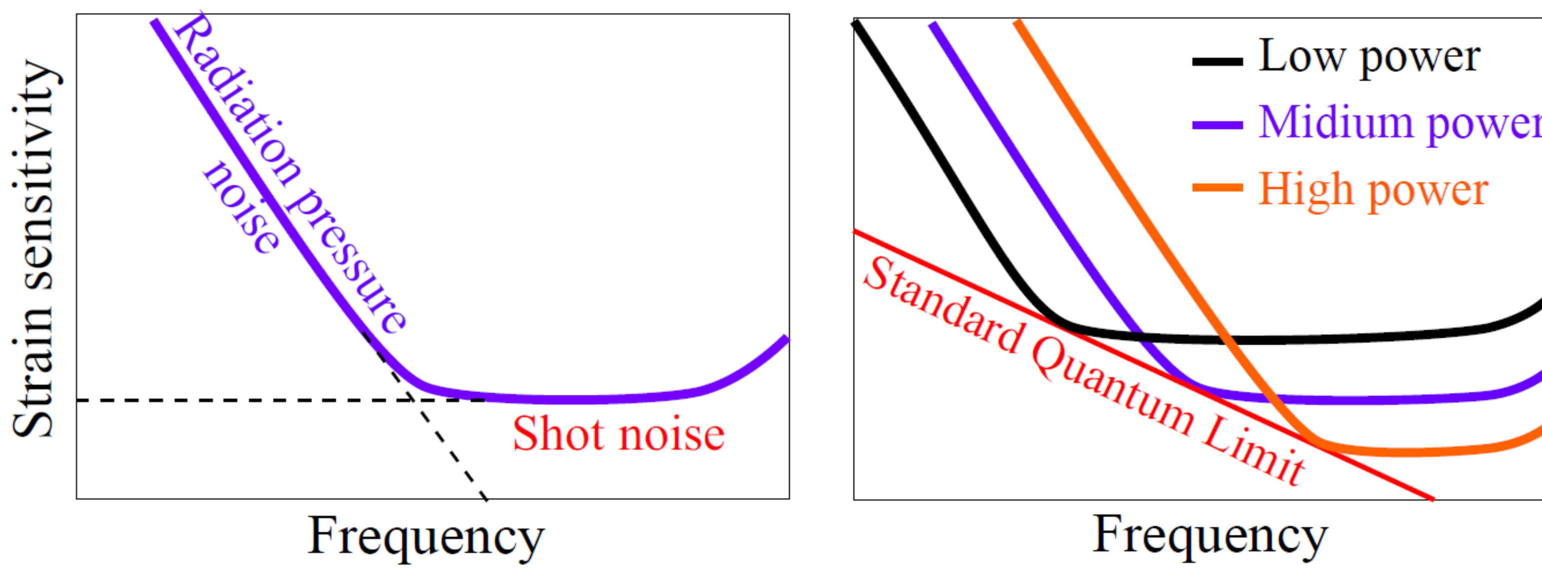
Quantum noise & Standard Quantum Limit



Quantum fluctuation in phase quadrature ⇨ **Shot noise**

In amplitude quadrature ⇨ **Power fluctuation** ⇨ **Radiation pressure noise**

Standard Quantum Limit:



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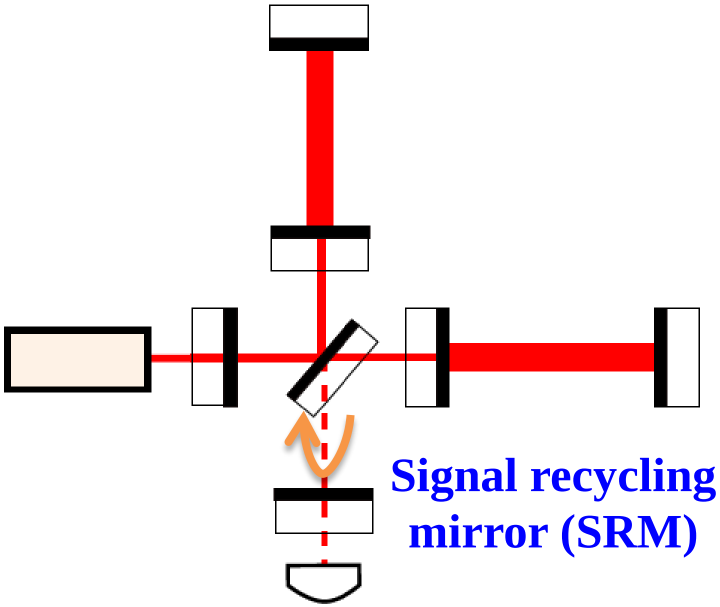
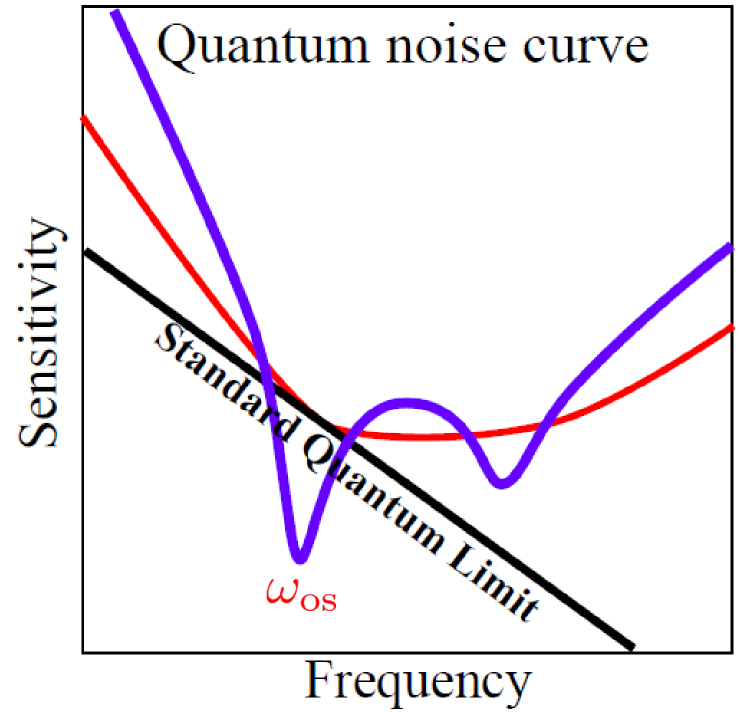
Detuned signal recycling (SR)



detuning $\neq 0, \pi/2$



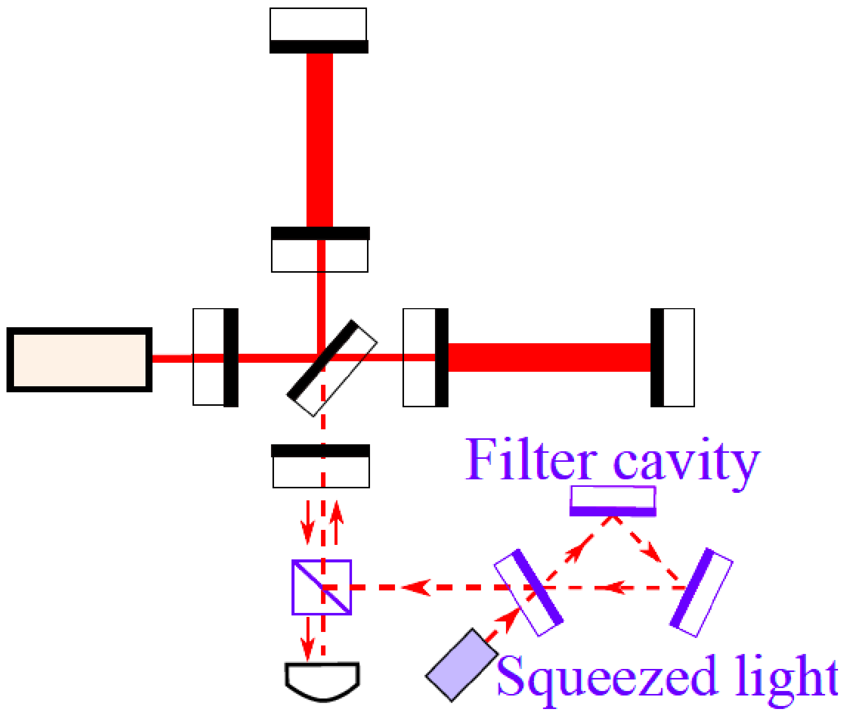
One interpretation [1]: optical spring effect ("self-force")

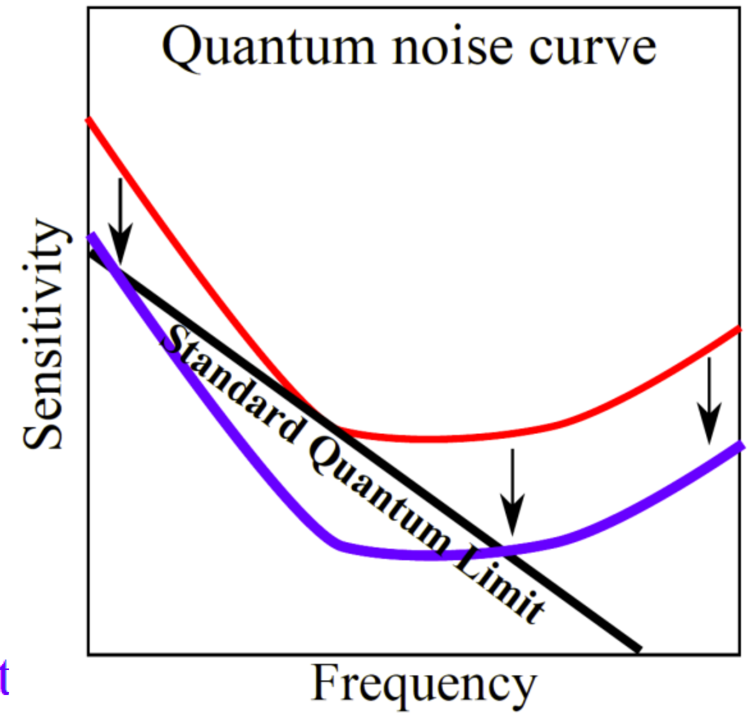
Time domain $M\ddot{x}(t) = F_{\rm GW}(t)$ $M\ddot{x}(t) = -M\omega_{\rm os}^2 x(t) + F_{\rm GW}(t)$

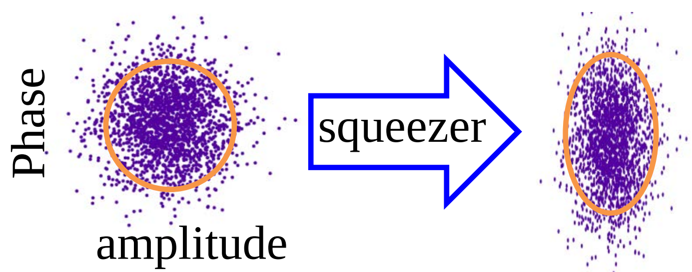
Frequency domain $R_{xx}(\Omega) = -1/(M\Omega^2)$ $R_{xx}(\Omega) = -1/[M(\Omega^2 - \omega_{\rm os}^2)]$

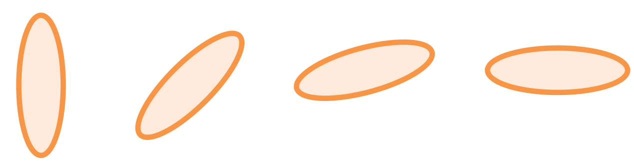
[1] A. Buonanno and Y. Chen. *Signal recycled laser-interferometer gravitational-wave detectors as optical springs*, PRD **65**, 042001 (2002).

Frequency-dependent squeezing



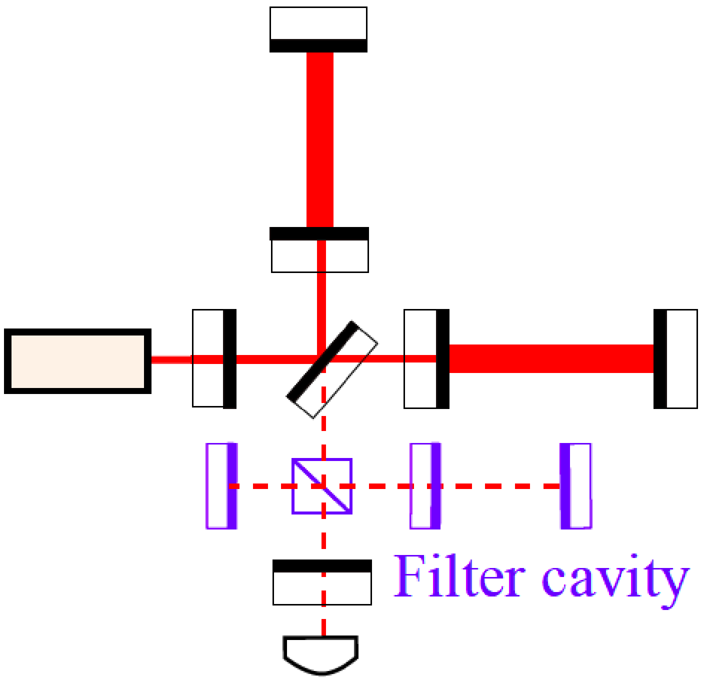


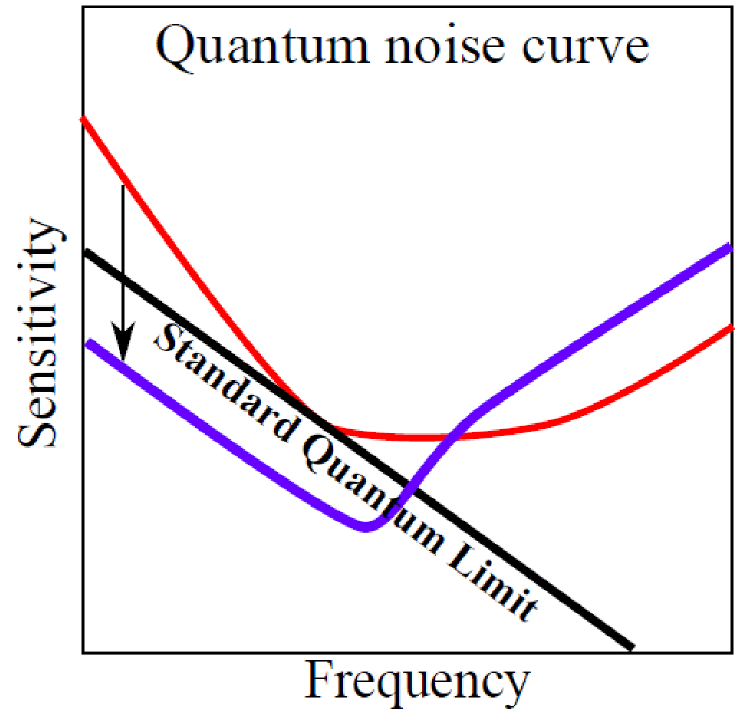




[1] J. Kimble, *et al. Conversion of conventional GW interferometers into QND by modifying their input and/or output optics*, PRD **65**, 022002 (2001)

Speed meter



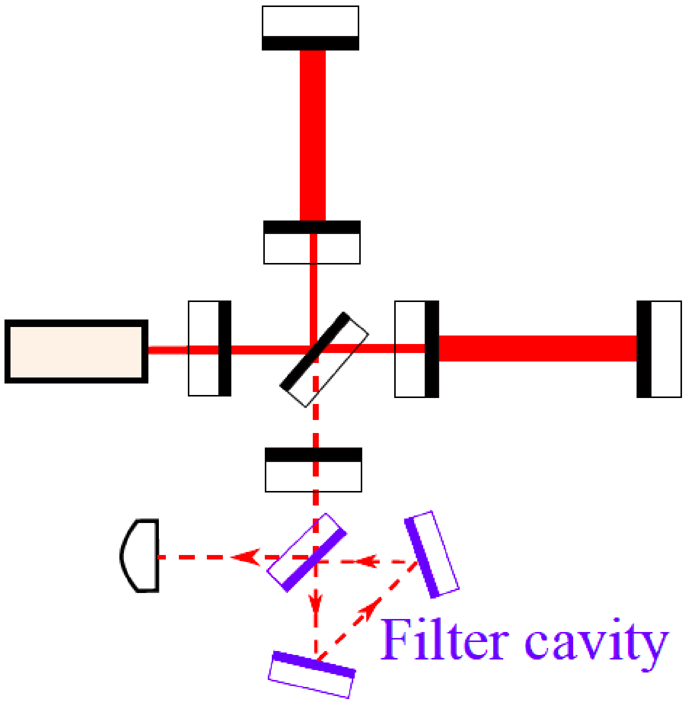


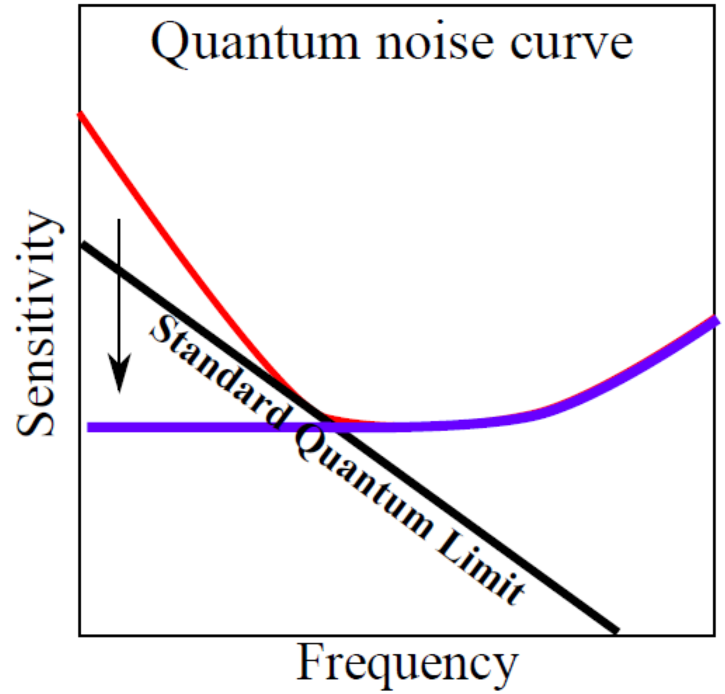
Interpretation: measuring conserved quantity—Momentum

Equivalent view: cancelling radiation pressure (back action) noise

[1] P. Purdue, and Y. Chen. *Practical speed meter designs for quantum nondemolition gravitational-wave interferometers*, PRD **67**, 122004 (2002).
[2] Y. Chen, *Sagnac interferometer as a speed-meter-type, quantum-nondemolition gravitational-wave detector*, PRD **67**, 122004 (2003).

Frequency-dependent (variational) readout





Coherent cancelation of radiation pressure noise by measuring proper quadrature at different frequencies [1] or using "negative mass" [2, 3].

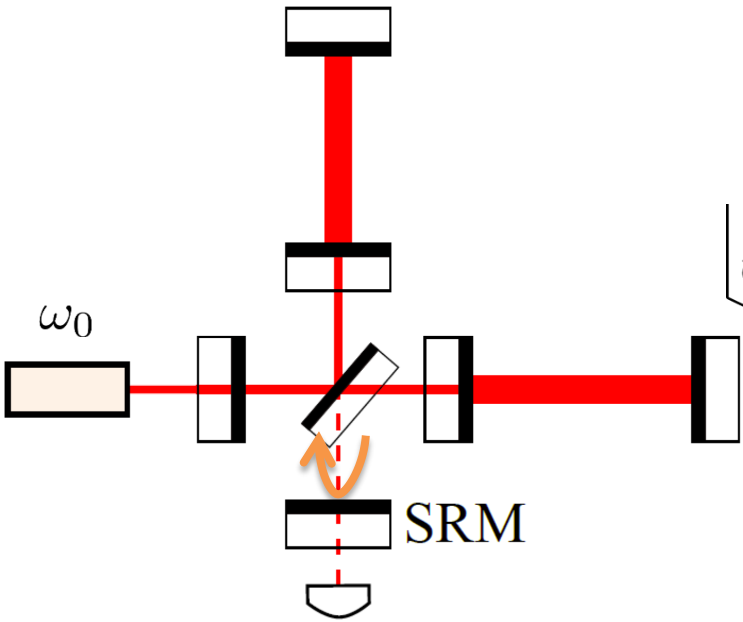
[1] J. Kimble *et al.*, PRD **65**, 022002 (2001).

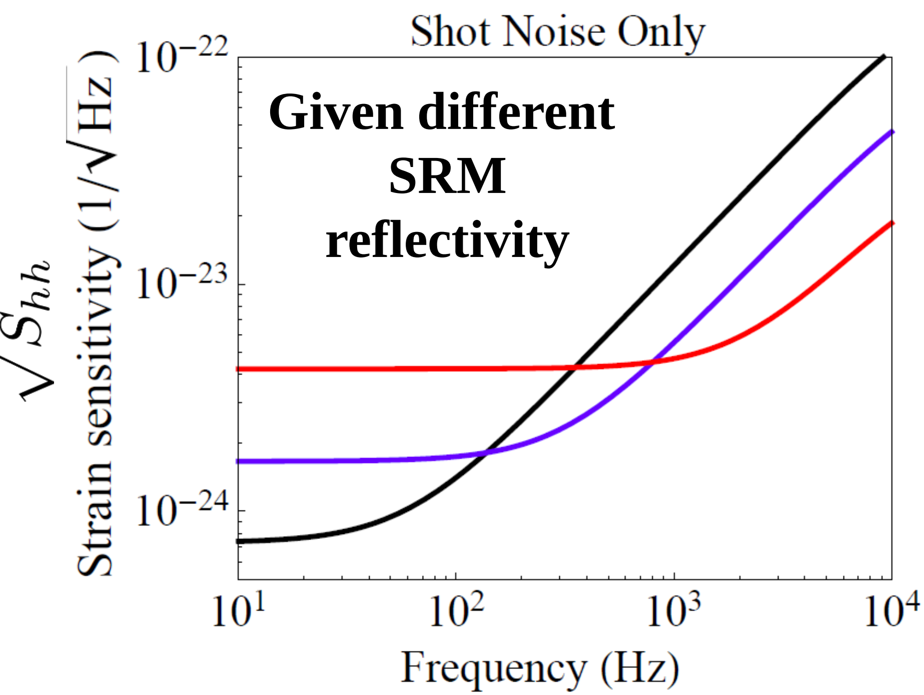
[2] M. Tsang, and C. Caves, *Coherent Quantum-Noise Cancellation for Optomechanical Sensors*, PRL **105**, 123601 (2010).

[3] M. Wimmer *et al.*, *Coherent cancellation of backaction noise in optomechanical force measurements*, PRA **89**, 053836 (2014).

Shot-noise-only sensitivity

Peak sensitivity and bandwidth tradeoff





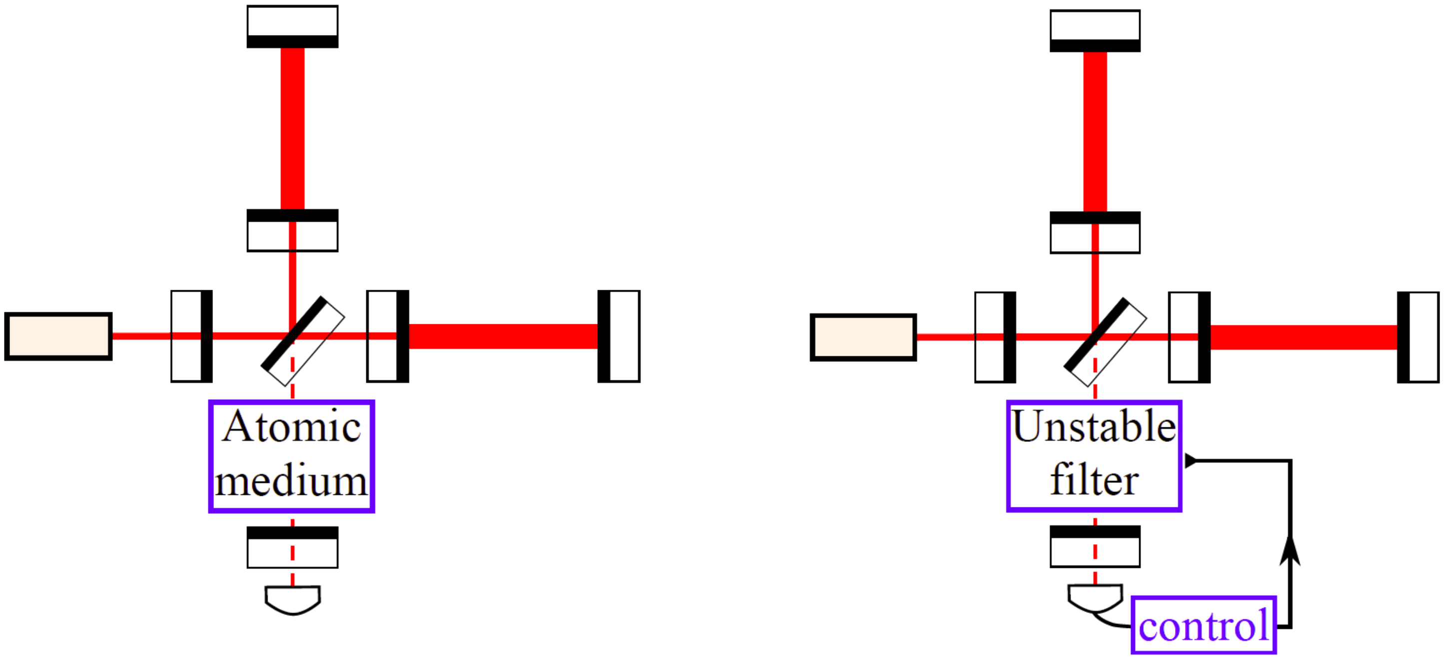
Signal resonant condition: $2\omega_0 L_{\rm arm}/c = 2n\pi$ (only at one frequency)

For frequency around it: $\omega = \omega_0 + \Omega$

$\Omega L_{\rm arm}/c = \pi/2$ ⇨ Feedback changes sign

Improving sensitivity-bandwidth product

White-light-cavity idea using atomic medium or unstable filter:

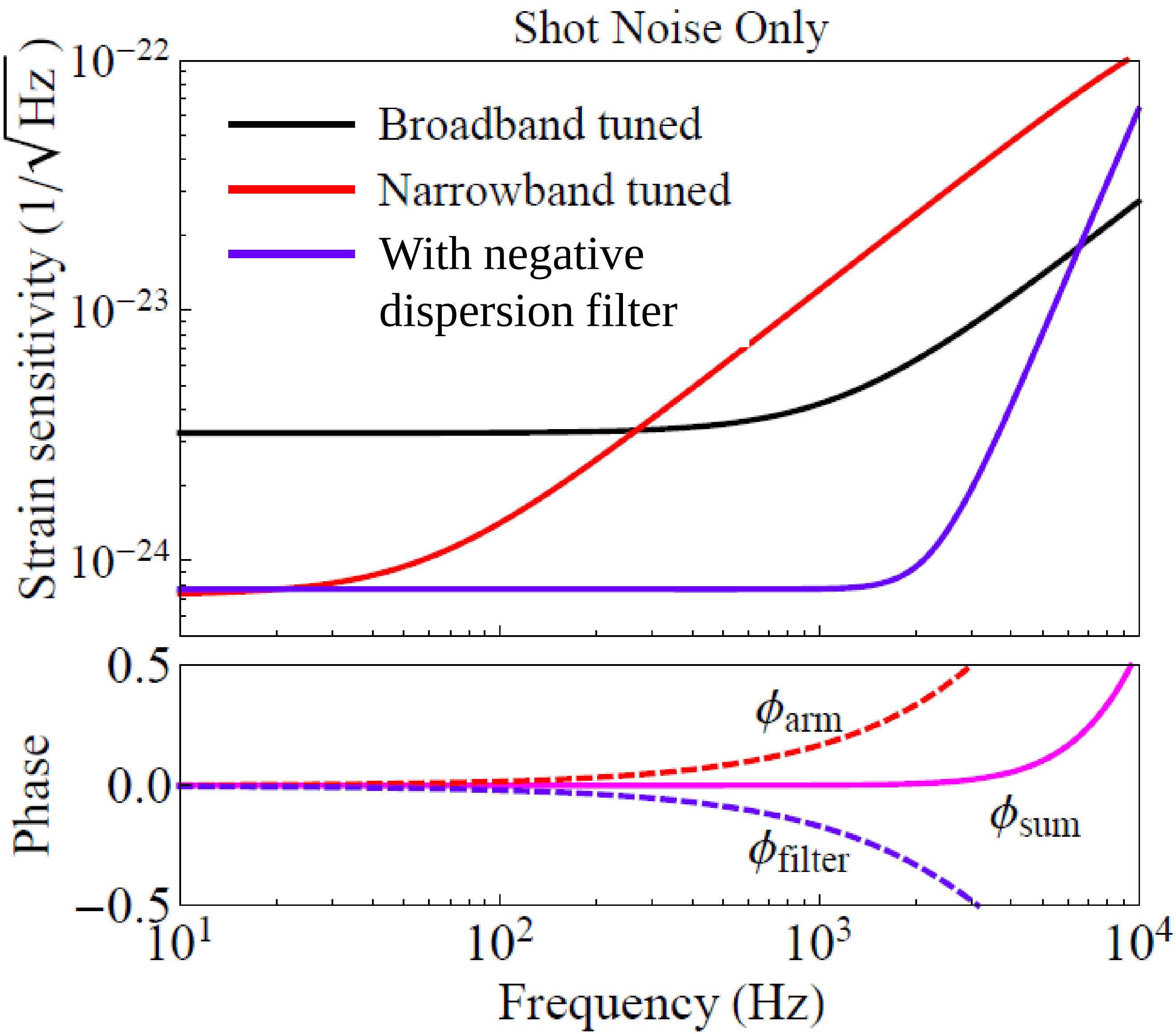


Round-trip phase: $2\omega L_{\rm arm}/c + 2\phi(\omega)$

Ideally: $\phi(\omega) = -\omega L_{\rm arm}/c$ or $\left.\frac{d\phi(\omega)}{d\omega}\right|_{\omega_0} = -\frac{L_{\rm arm}}{c} < 0$ **Negative dispersion**

Resonant condition is satisfied for broad frequency band

Resulting shot-noise limited sensitivity



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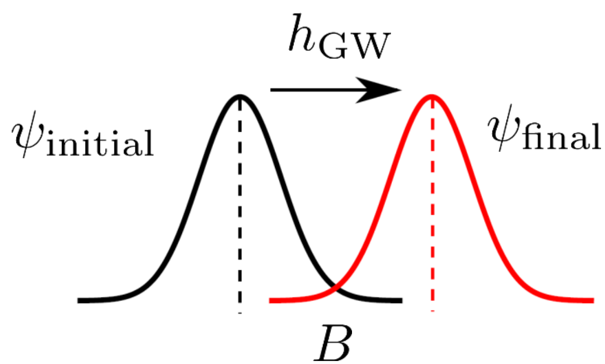
Fundamental Quantum Limit

Detecting GW as parameter estimation:

Initial quantum state of detector $\psi_{\rm initial}$ → $\boxed{\hat{H}_{\rm int}(h_{\rm GW})}$ (Interaction) → Final state $\psi_{\rm final}(h_{\rm GW})$



$$\hat{H}_{\rm int} = h_{\rm GW}\hat{A}$$

$$[\hat{A},\, \hat{B}] = i\hbar$$

Minimal detectable signal depending on distinguishability of two states

In terms of quantum uncertainty: $h^2_{\rm GW}|_{\rm min} \propto \sigma_{BB} \geq \hbar^2/\sigma_{AA}$

⇨ **Quantum counterpart of the classical Cramér-Rao bound**

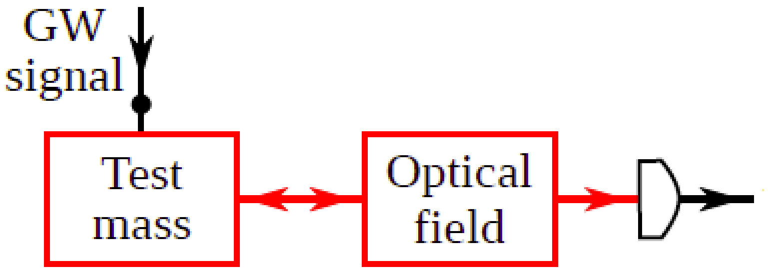
[1] C. Helstrom Physics Letters A 25, 101 (1967).
[2] A. Holevo, *Probabilistic and Statistical Aspects of Quantum Theory* (1982).
[3] S. L. Braunstein and C. M. Caves, Phys. Rev. Lett. 72, 3439 (1994).

Fundamental Quantum Limit

Two equivalent pictures:

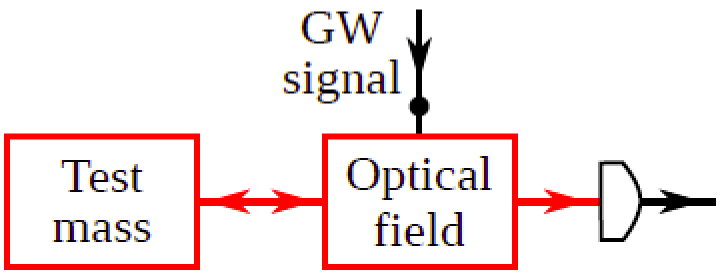
$$\hat{H}_{\rm int}|_{\rm LIF} = M L_{\rm arm} \ddot{h}_{\rm GW}\, \hat{X}_{\rm TM}$$

(Local Inertial Frame)



$$\hat{H}_{\rm int}|_{\rm TT} = 2\, h_{\rm GW} \hat{P}_{\rm arm} L_{\rm arm}/c$$

(TT Frame)



In terms of noise spectrum, both give: $S_{hh}(\Omega) \geq \dfrac{\hbar^2 c^2}{4L_{\rm arm}^2 S_{PP}(\Omega)}$

[1] V. Braginsky, M. Gorodetsky, F. Khalili, and K. Thorne, *Energetic quantum limit in large-scale interferometers*, AIP Conference Proceedings **523**, 180 (2000).

[2] M. Tsang, H. Wiseman, and C. Caves, *Fundamental Quantum Limit to Waveform Estimation*, Phys. Rev. Lett. **106**, 090401 (2011).

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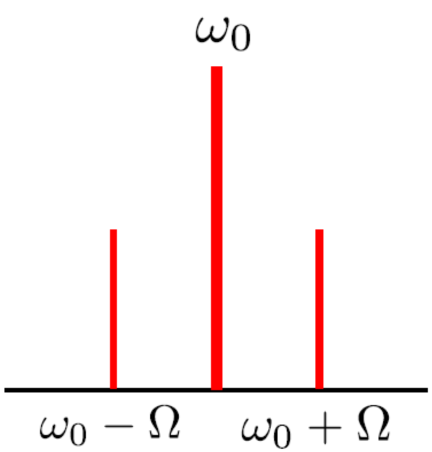
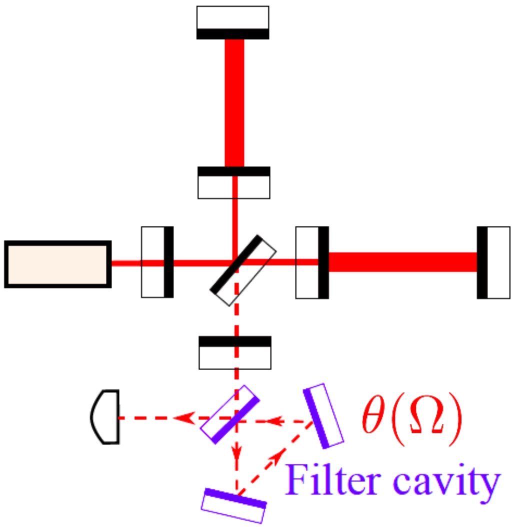
Condition for achieving the minimum

The minimum:

$$S_{hh}(\Omega)|_{\min} = \frac{\hbar^2 c^2}{4L_{\rm arm}^2 S_{PP}(\Omega)}$$

General condition for achieving it:





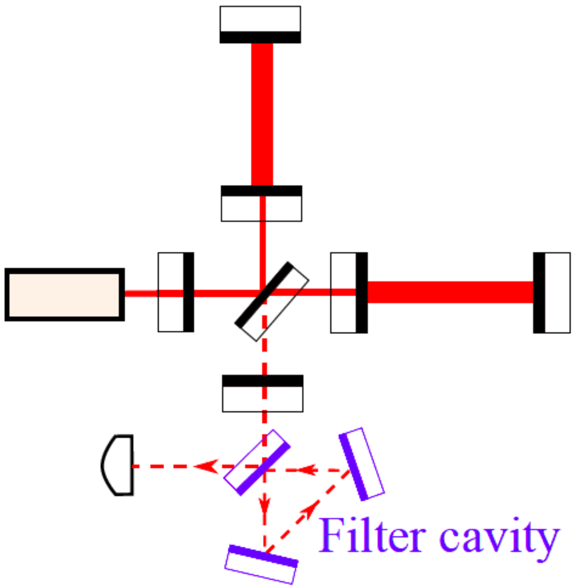
1. Upper & lower sidebands contribute equally to power fluctuation. What if not?

2. Homodyne detection of proper output quadrature.

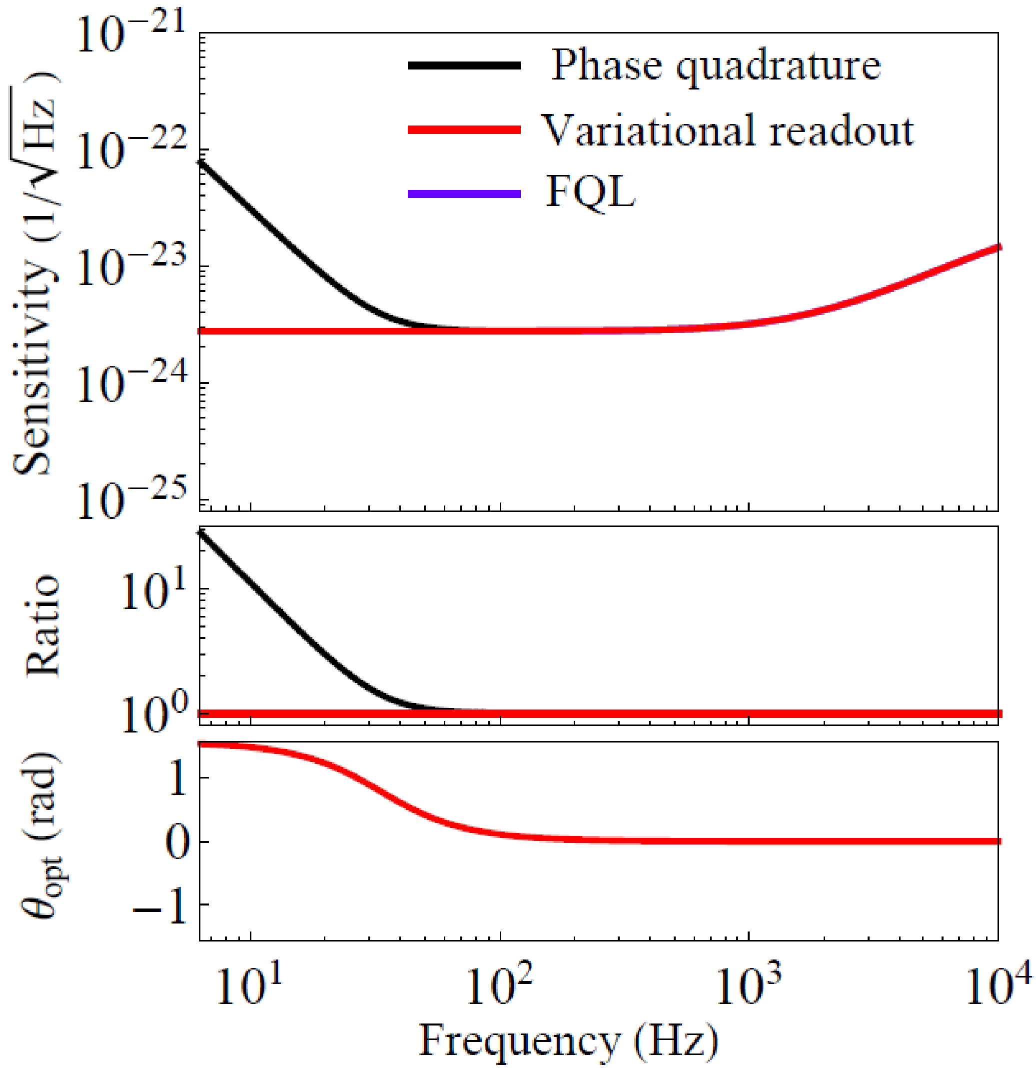
No optical loss (loss issue is under study)

[1] H. Miao, R. Adhikari, Y. Ma, and Y. Chen, LIGO-DCC: P1600092 (2016) [arXiv soon]

Example of tuned signal recycling

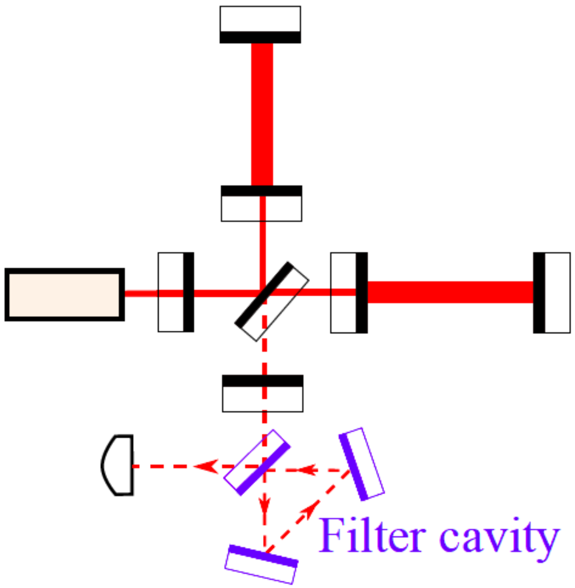


$$\text{detuning} = 0, \pi/2$$

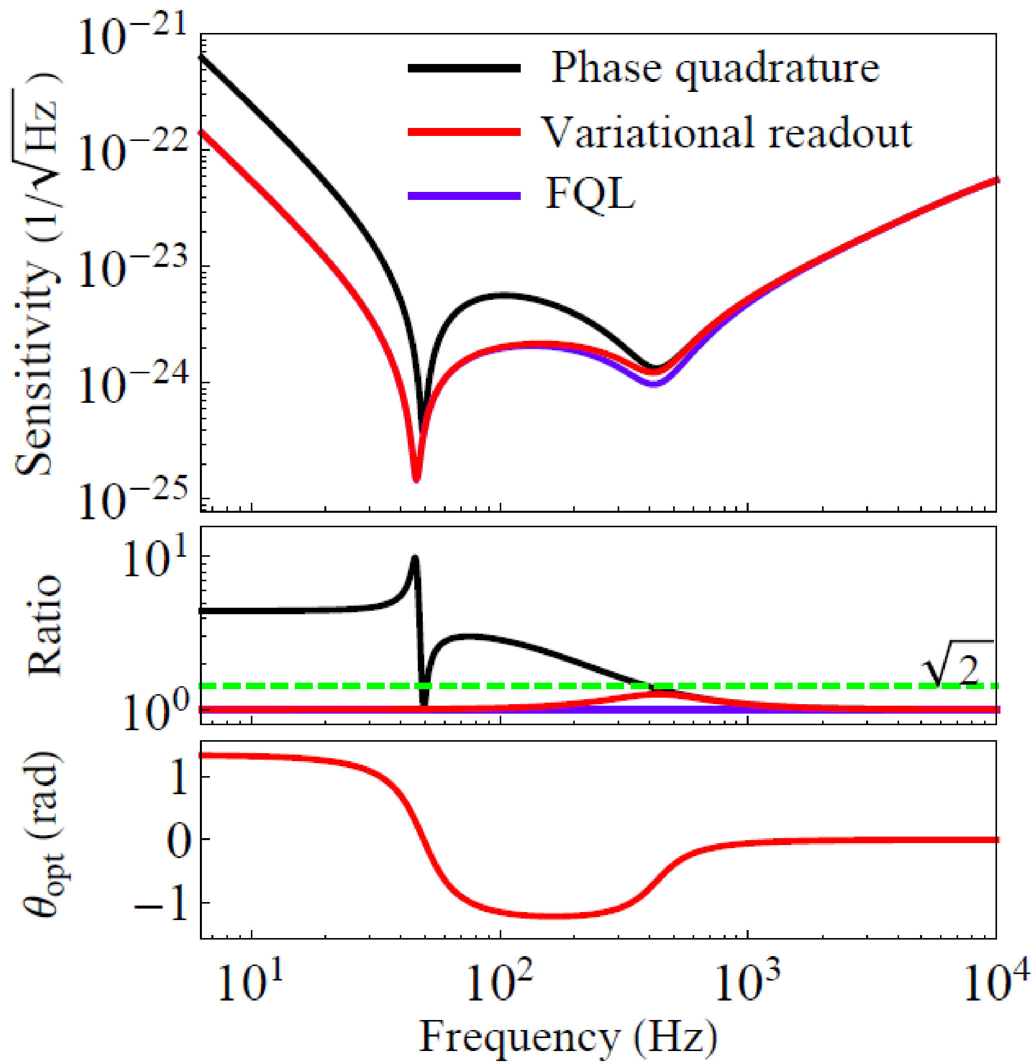


FQL is achieved by using frequency-dependent (variational) readout.

Example of detuned signal recycling



$\text{detuning} \neq 0, \pi/2$



FQL is close to that of frequency-dependent (variational) readout but not exact.

[1] H. Miao, R. Adhikari, Y. Ma, and Y. Chen, LIGO-DCC: P1600092 (2016)

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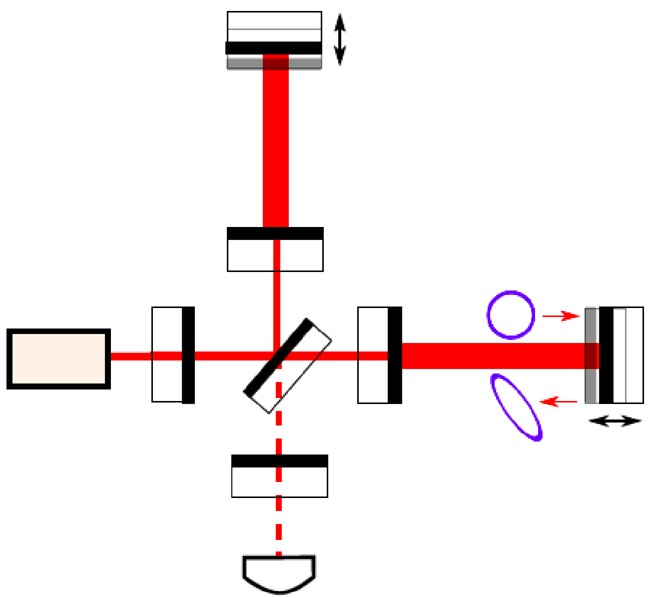
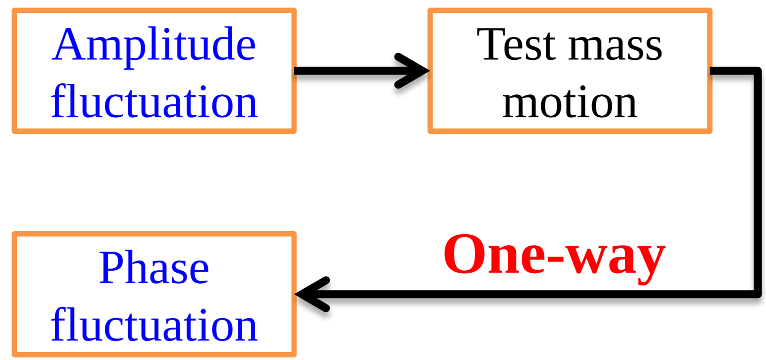
How to lower the fundamental limit

$$S_{hh}(\Omega)|_{\min} = \frac{\hbar^2 c^2}{4L_{\rm arm}^2 S_{PP}(\Omega)}$$

What determines $S_{PP}(\Omega)$

- **Optical power**
- **External phase squeezing if any**
- **Built-in ponderomotive squeezing!**

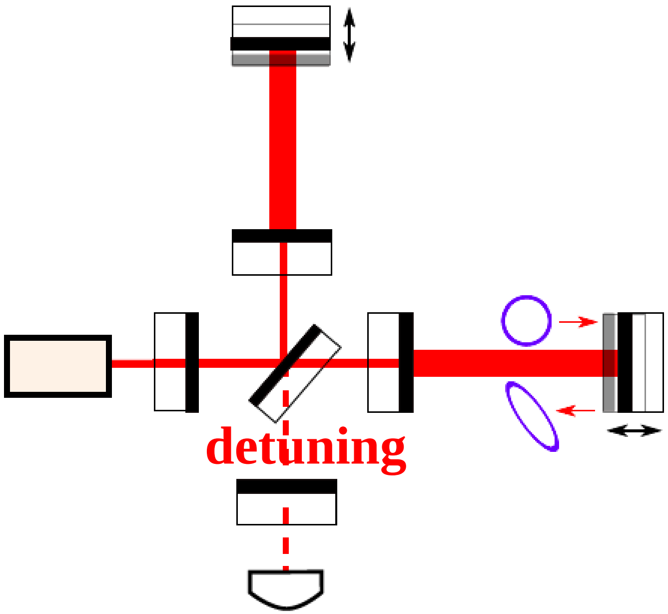


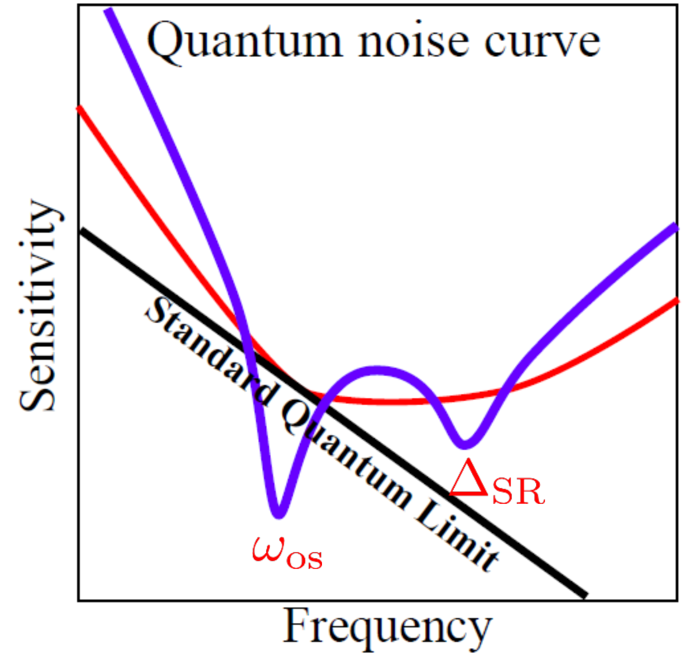


Usually does not modify power fluctuation
(i.e. amplitude fluctuation)

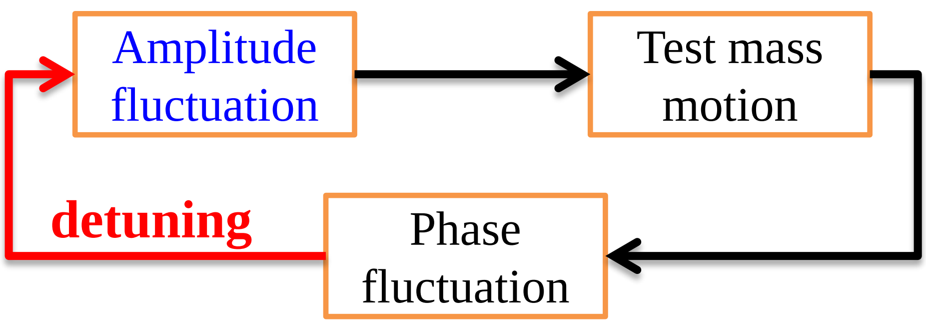
A new perspective on optical spring

Detuned signal recycling:





Coherent optical feedback:



$$G_{\rm CL}(\Omega) = \frac{1}{1 - G_{\rm OL}(\Omega)}$$

$$|G_{\rm OL}(\omega_{\rm os})| = 1$$

$S_{PP}(\omega_{\rm os})$ **is very large.**

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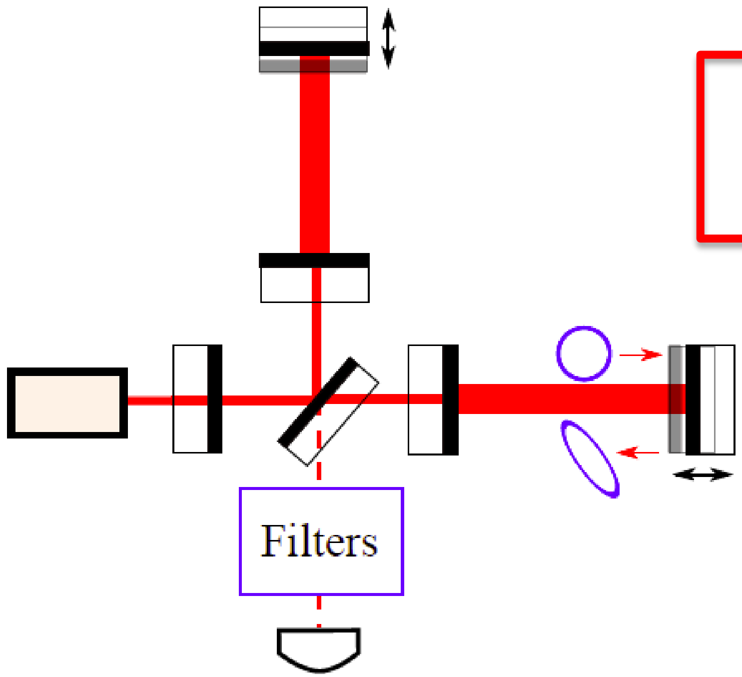
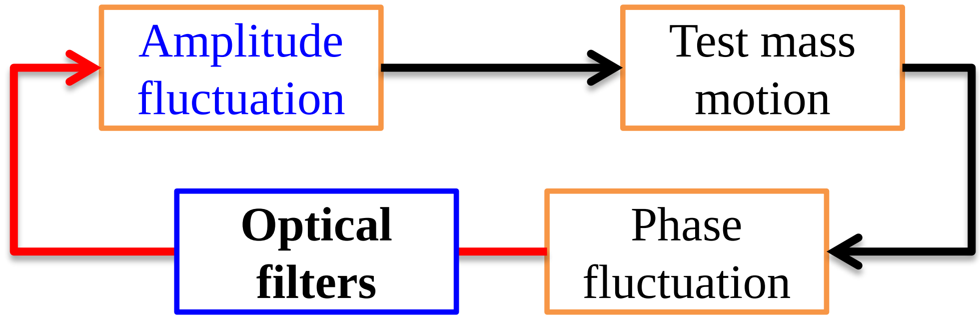
Coherent optical feedback

Include ponderomotive squeezing as a part of the design:





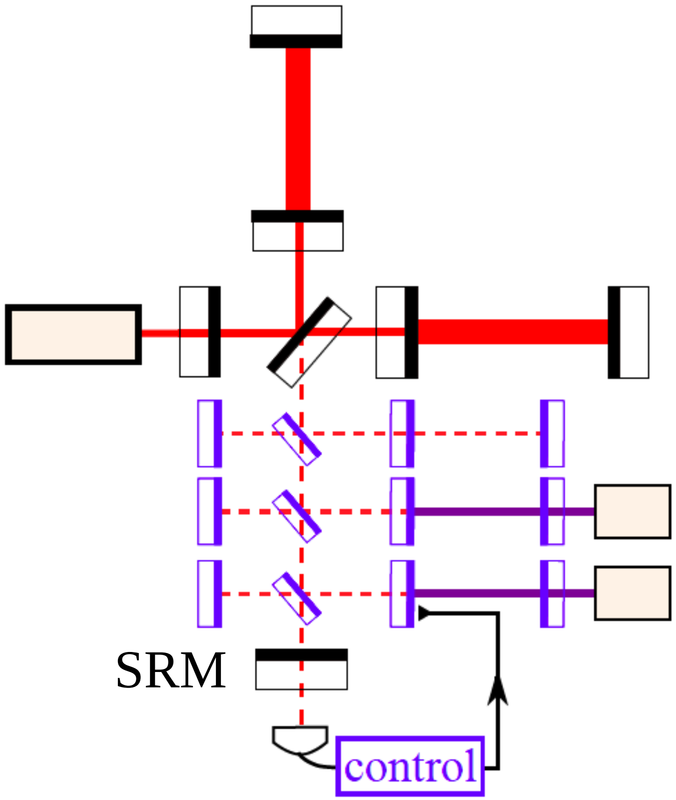
$$G_{\rm CL}(\Omega) = \frac{1}{1 - |G_{\rm OL}(\Omega)|e^{i\phi(\Omega)}}$$

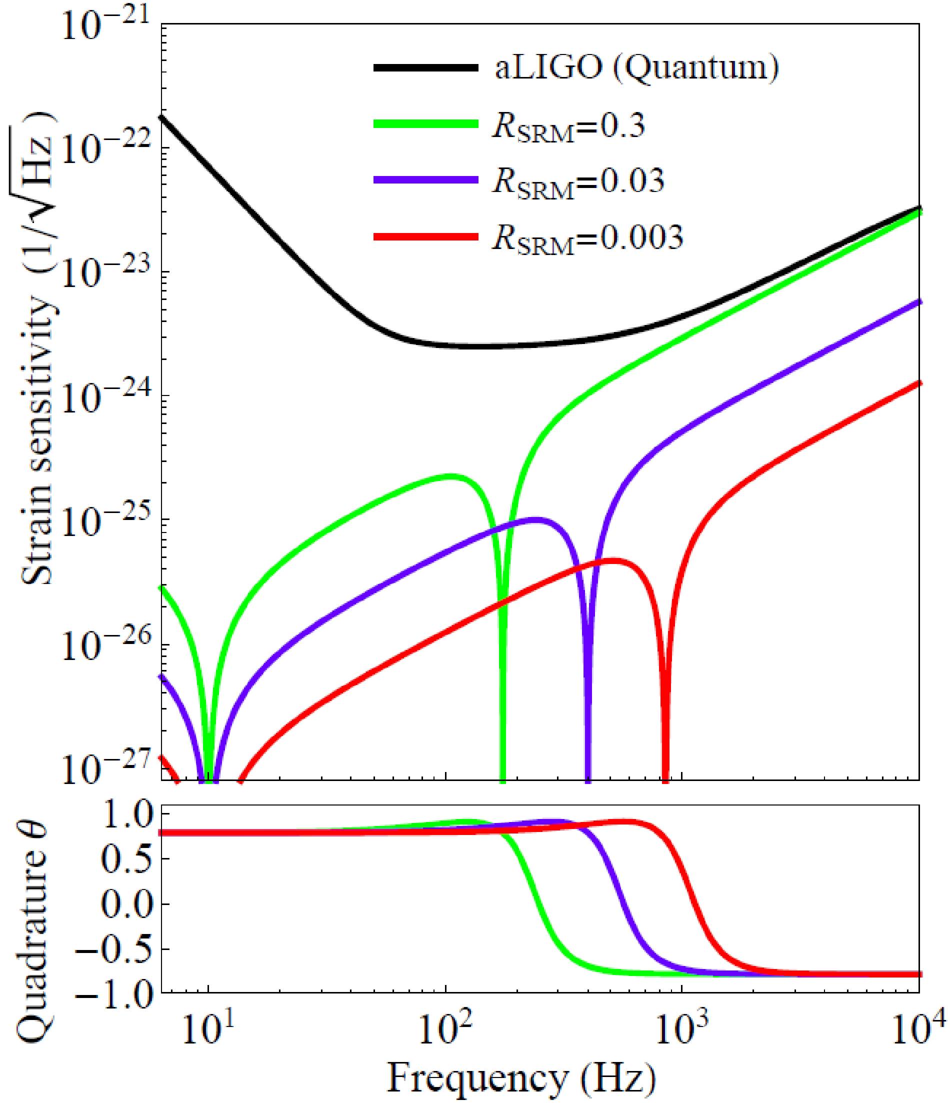
$$|G_{\rm OL}(\Omega)| \approx 1 \qquad \phi(\Omega) \approx 0$$

$S_{PP}(\Omega)$ **large over broad frequency**

$$S_{hh}(\Omega)|_{\rm min} = \frac{\hbar^2 c^2}{4L_{\rm arm}^2 S_{PP}(\Omega)} \longrightarrow$$ **“Zero” or “No” limit**

One example (preliminary)

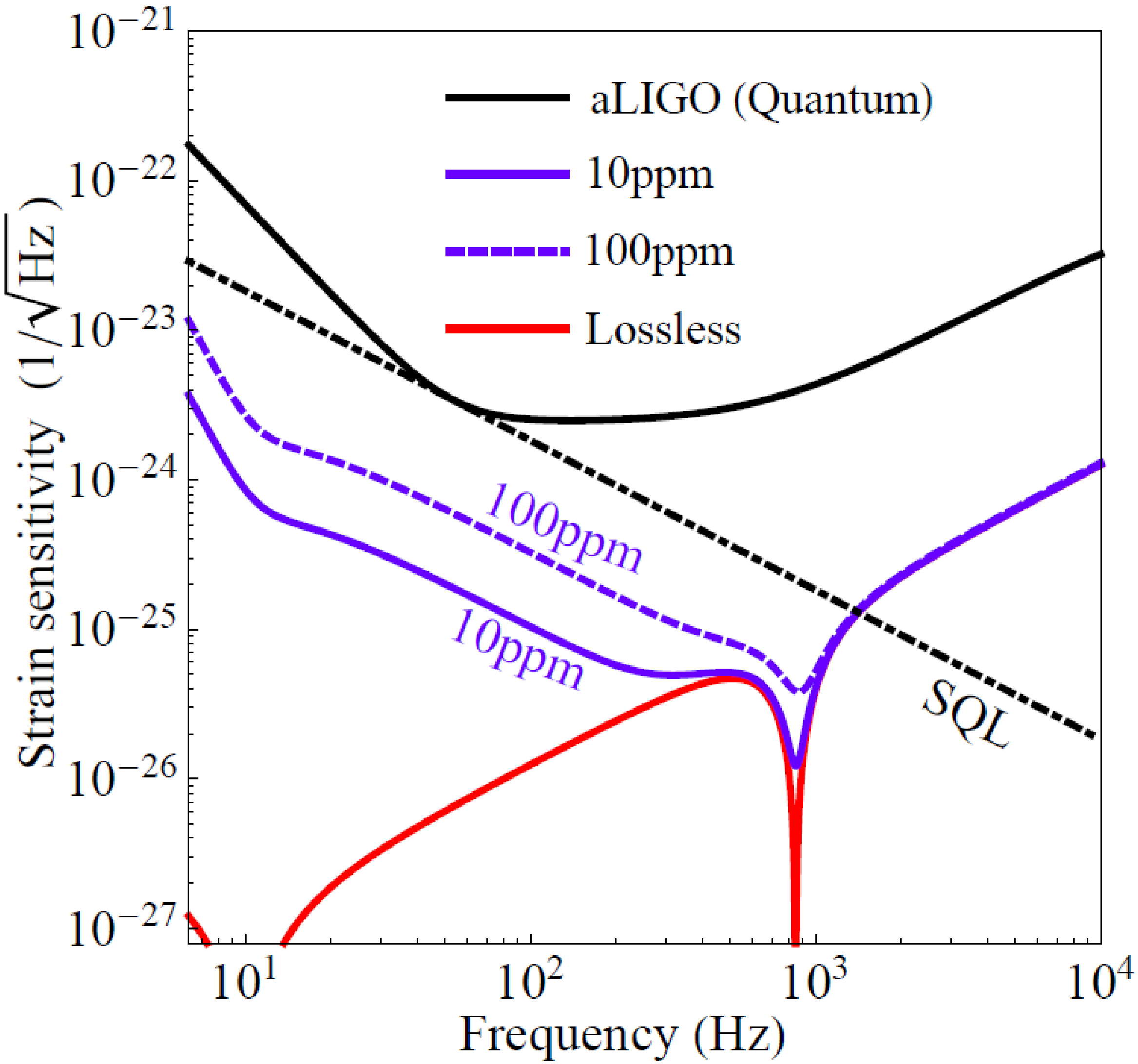




One passive filter to flatten the gain from pondermotive squeezing. Two unstable filters to compensate the phase.

Ideal lossless case

One example (preliminary)



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General condition for achieving it

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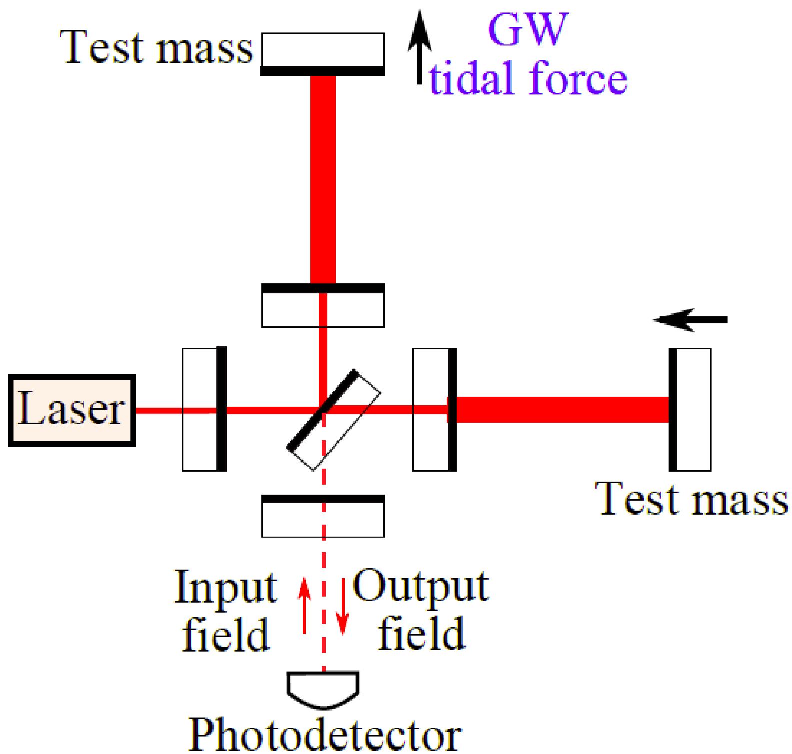
"No" limit in the lossless case (optical loss issue is under study)

Supplemental slides

Quantum noise modelling



Model: a continuous quantum measurement process

GW: classical tidal force

Test mass: quantum harmonic oscillator

Optical field: quantum field

Photodetector: projective measurement

Quantum noise modelling

Quantization of test mass (center-of-mass motion):

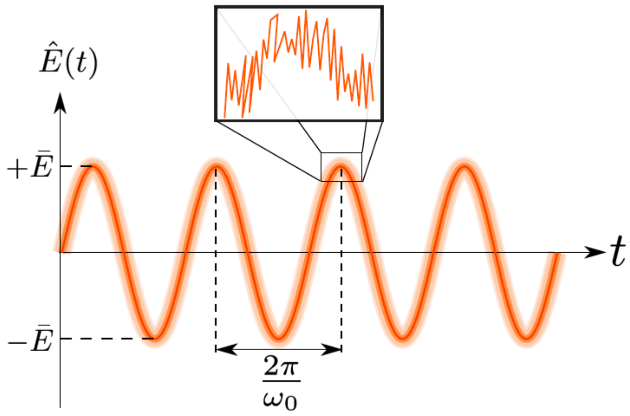
A quantum harmonic oscillator (~ a free mass)

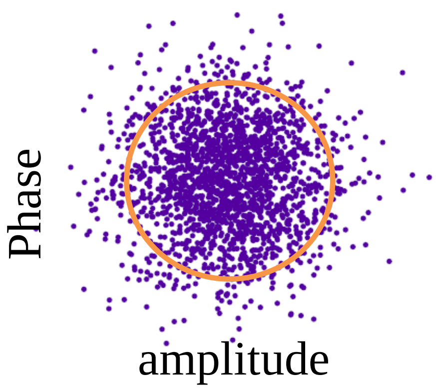
Quantization of optical field:

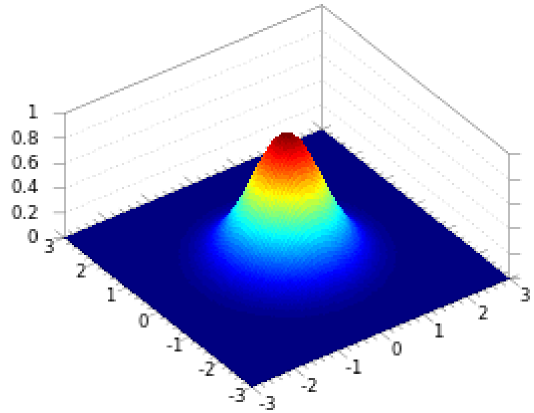
Field “position”: Phase quadrature

Field “momentum”: Amplitude quadrature

Satisfying
Heisenberg Uncertainty
Relation

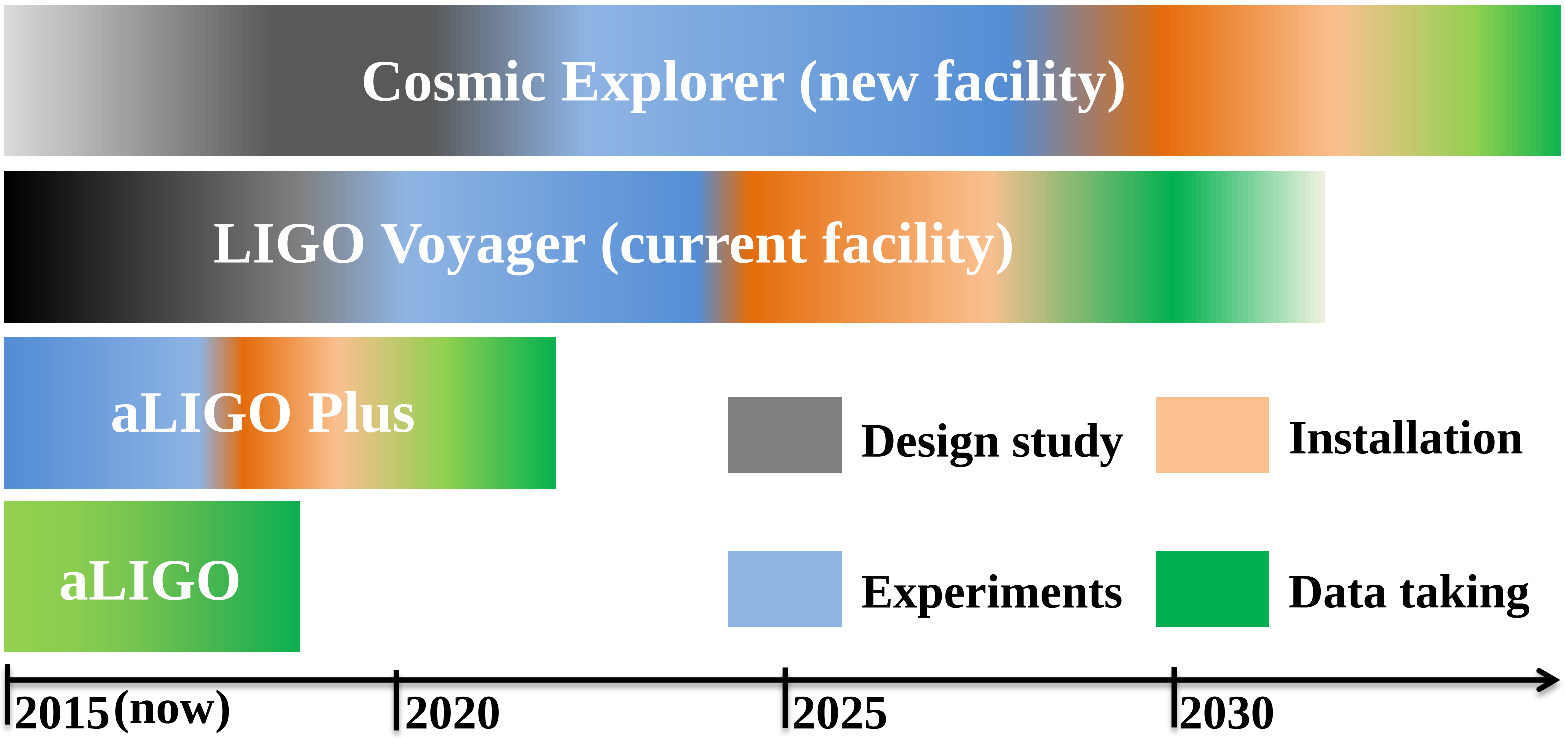






Beyond Advanced LIGO

Future upgrades



Reference:

LIGO Scientific Collaboration, Instrument Science White Paper (2014-2015)

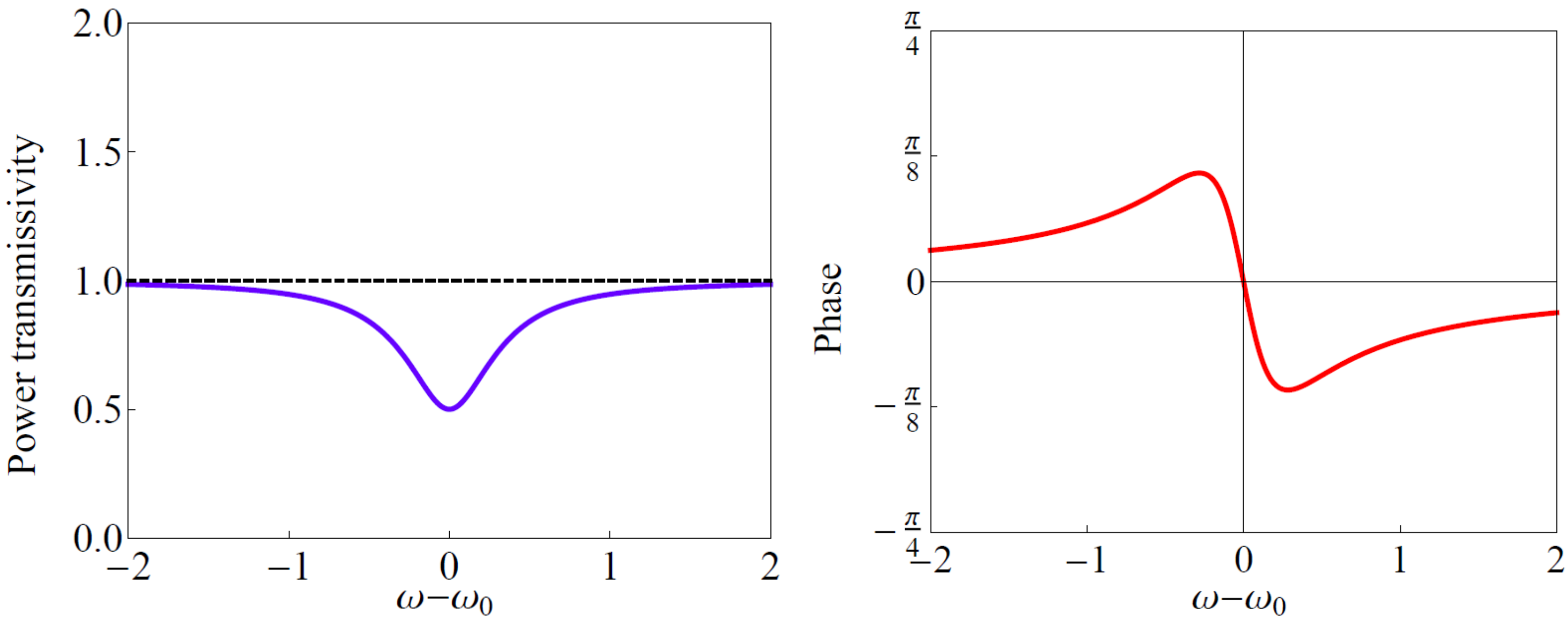
White-light-cavity using atomic medium

Dispersion of medium:

For usual mediums at low frequencies:

$$\left.\frac{\mathrm{d}\phi}{\mathrm{d}\omega}\right|_{\text{low freq}} > 0 \quad \Rightarrow \quad \text{positive (normal) dispersion}$$

Around absorption (attenuation) line (Kramers-Kronig relation):



White-light-cavity using atomic medium

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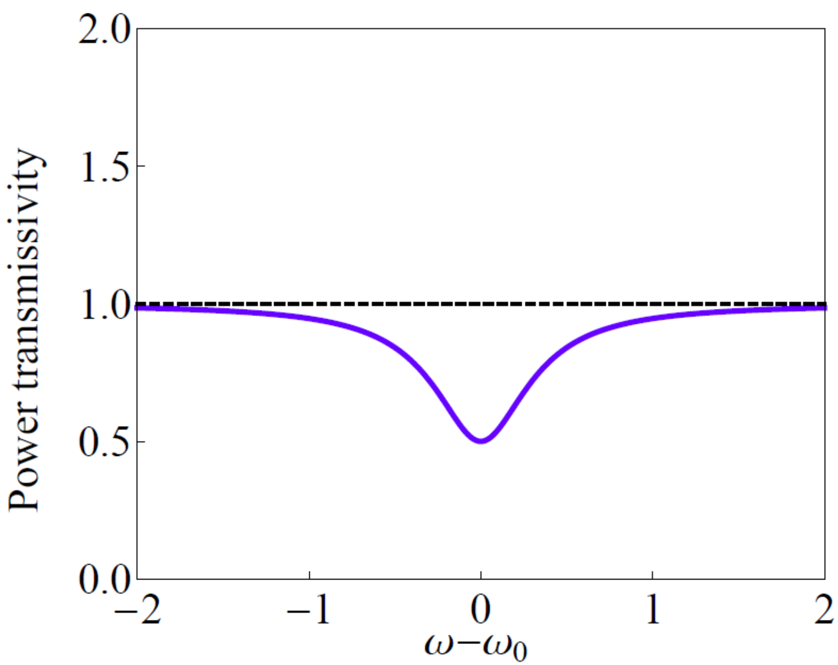
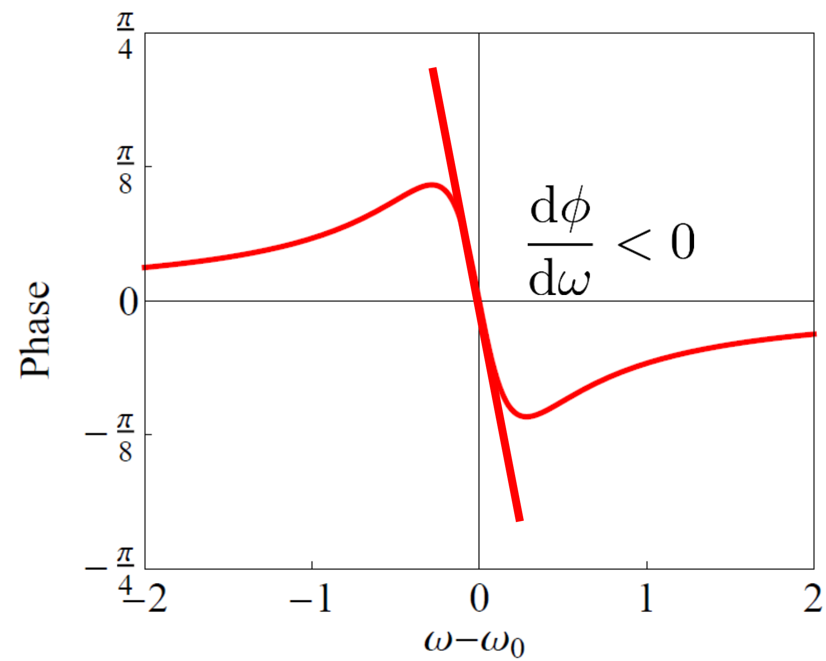
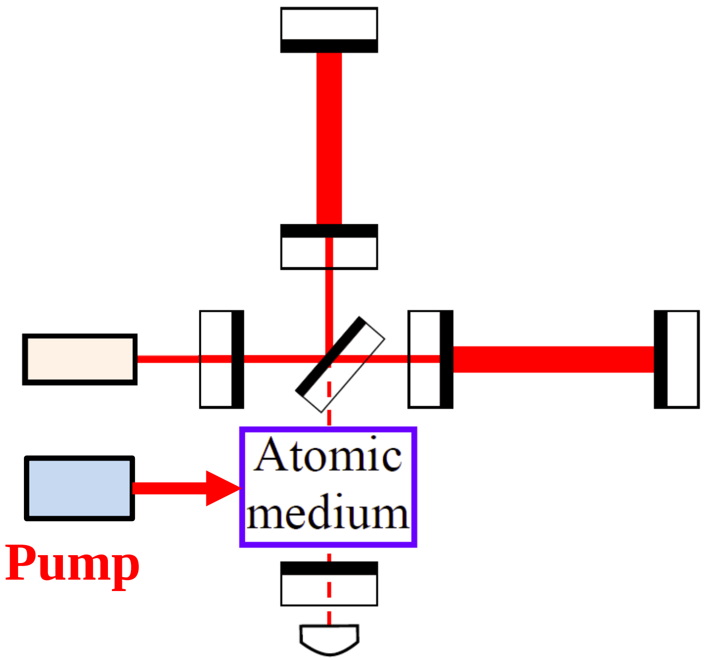


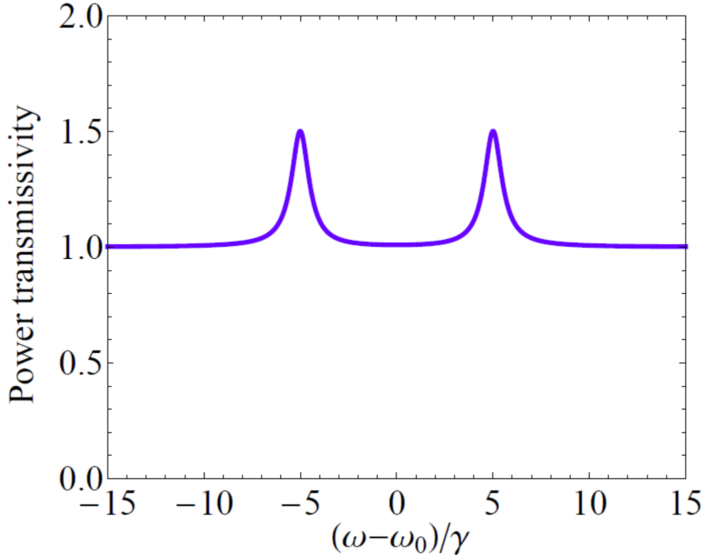


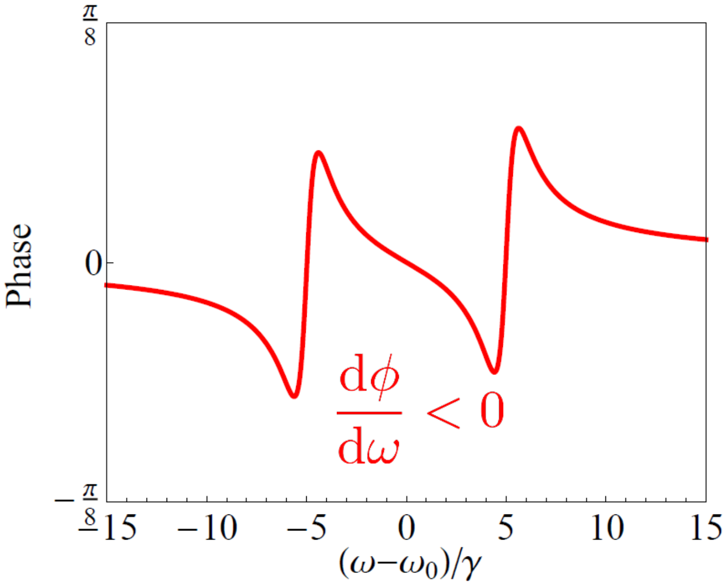
v

White-light-cavity using atomic medium

Instead using active gain medium:





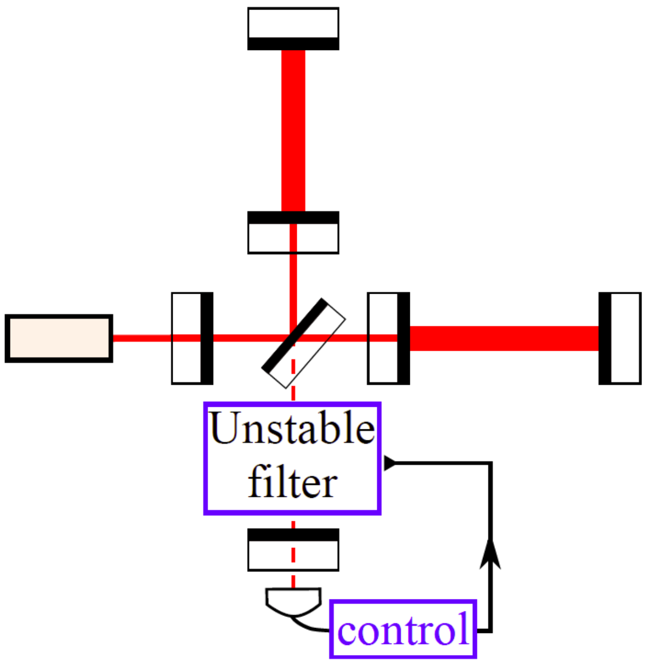


[1] A. Wicht *et al.*, *White-light cavities, atomic phase coherence, and GW detectors*, Optics Communications (2000).

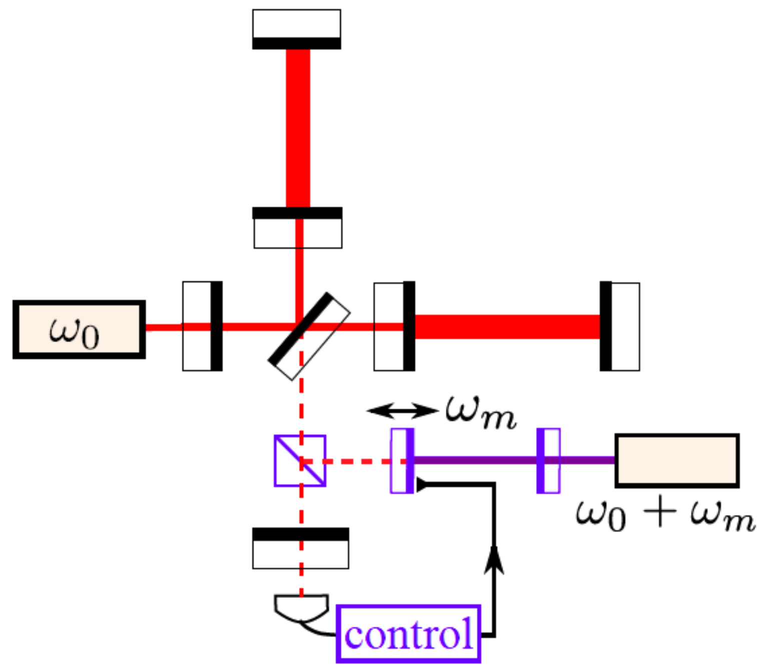
[2] M. Zhou, Z. Zhou, and S. M. Shahriar. *Quantum noise limits in white-light-cavity enhanced gravitational wave detectors.* Phys. Rev. D **92**, 082002 (2015).

White-light-cavity using unstable filter

General cases:



An example[1]:

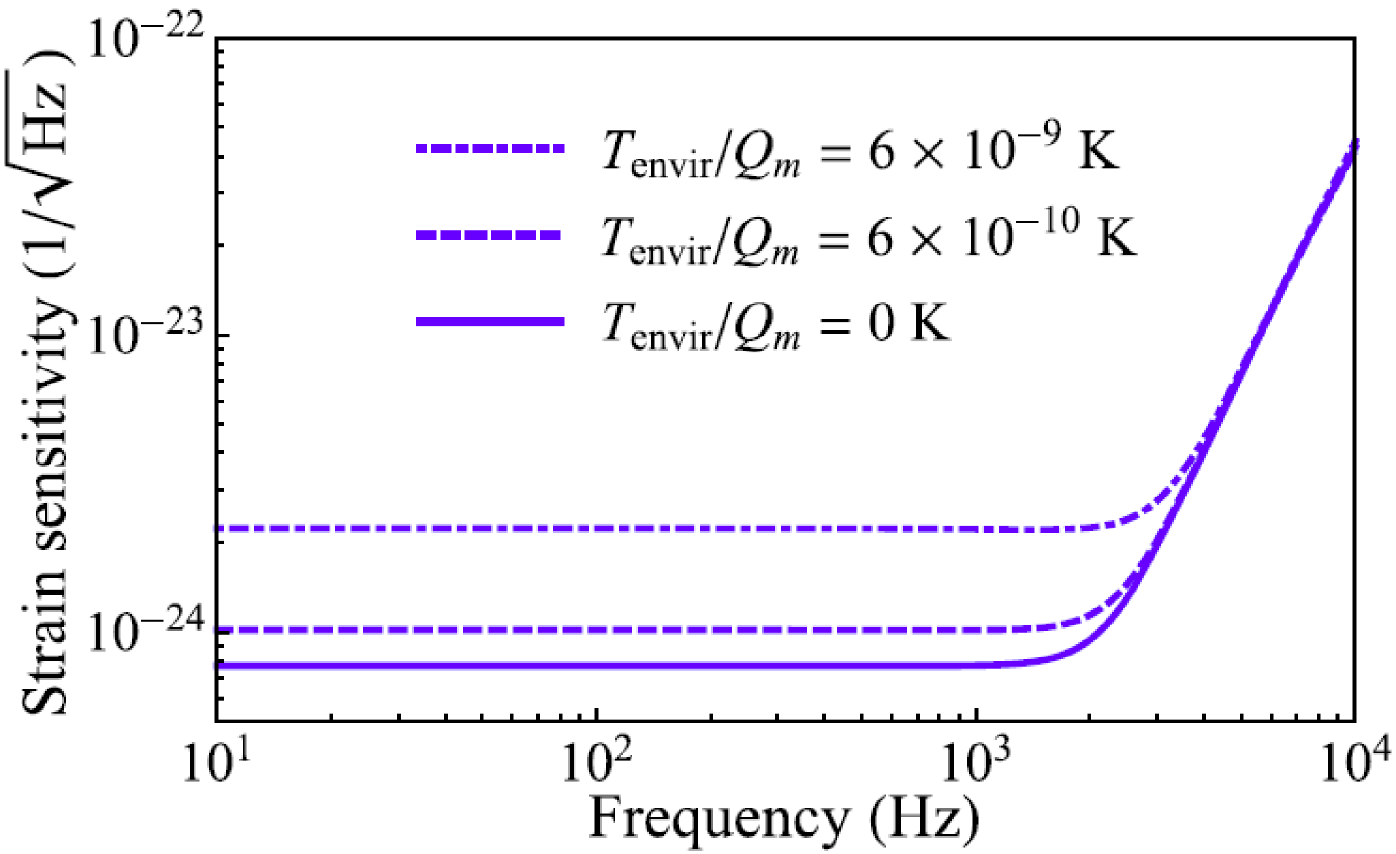


$|T(\omega)| \approx 1$ $\quad \mathrm{d}\phi/\mathrm{d}\omega < 0$ **(not constrained by Kramers-Kronig relation)**

[1] H. Miao, Y. Ma, C. Zhao and Y. Chen, *Enhancing the bandwidth of gravitational wave detectors with unstable optomechanical filters*, PRL **115**, 211104 (2015)

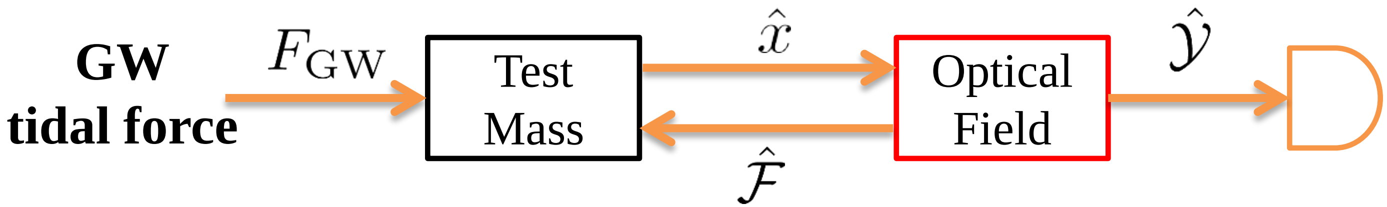
Unstable optomechanical filter

Thermal noise issue:



Fundamental Quantum Limit

Derivation using linear quantum measurement theory:



$$\hat{\mathcal{Y}}(\Omega) = \hat{\mathcal{Z}}(\Omega) + \hat{x}(\Omega) \qquad \hat{x}(\Omega) = R_{xx}(\Omega)[\hat{\mathcal{F}}(\Omega) + F_{\rm GW}(\Omega)]$$

Shot noise **Radiation pressure noise**

h-referred noise spectrum ($R_{xx} = -1/(M\Omega^2)$ $F_{\rm GW} = ML_{\rm arm}h(\Omega)\Omega^2$):

$$S_{hh}(\Omega) = \frac{1}{L_{\rm arm}^2}\left[S_{\mathcal{ZZ}}(\Omega) - \frac{2}{M\Omega^2}S_{\mathcal{ZF}}(\Omega) + \frac{1}{M^2\Omega^4}S_{\mathcal{FF}}(\Omega)\right]$$

Correlation between two noises

Heisenberg Uncertainty Principle:

$$S_{\mathcal{ZZ}}(\Omega)S_{\mathcal{FF}}(\Omega) - S_{\mathcal{ZF}}^2(\Omega) \geq \hbar^2$$

Fundamental Quantum Limit

No correlation $S_{\mathcal{Z}\mathcal{F}} = 0$ **: Standard Quantum Limit (SQL)**

$$S_{hh}(\Omega) = \frac{1}{L_{\rm arm}^2}\left[S_{\mathcal{Z}\mathcal{Z}}(\Omega) + \frac{1}{M^2\Omega^4}S_{\mathcal{F}\mathcal{F}}(\Omega)\right] \geq \frac{2\hbar}{M\Omega^2 L_{\rm arm}^2} \equiv S_{hh}^{\rm SQL}(\Omega)$$

With correlation $S_{\mathcal{Z}\mathcal{F}} \neq 0$**: Fundamental Quantum Limit**

Heisenberg Uncertainty Principle: $S_{\mathcal{Z}\mathcal{Z}}(\Omega)S_{\mathcal{F}\mathcal{F}}(\Omega) - S_{\mathcal{Z}\mathcal{F}}^2(\Omega) \geq \hbar^2$

$$S_{\mathcal{Z}\mathcal{Z}}(\Omega) \geq [\hbar^2 + S_{\mathcal{Z}\mathcal{F}}^2(\Omega)]/S_{\mathcal{F}\mathcal{F}}(\Omega)$$

$$S_{hh}(\Omega) \geq \frac{\hbar^2}{L_{\rm arm}^2 S_{\mathcal{F}\mathcal{F}}(\Omega)} + \frac{1}{S_{\mathcal{F}\mathcal{F}}(\Omega)}\left[S_{\mathcal{Z}\mathcal{F}}(\Omega) - \frac{1}{M\Omega^2}S_{\mathcal{F}\mathcal{F}}(\Omega)\right]^2$$

$$\geq \frac{\hbar^2}{L_{\rm arm}^2 S_{\mathcal{F}\mathcal{F}}(\Omega)} = \frac{\hbar^2 c^2}{4L_{\rm arm}^2 S_{PP}(\Omega)}$$